A BLIND FREQUENCY DOMAIN METHOD FOR DS-CDMA SYNCHRONIZATION USING ANTENNA ARRAYS

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ABSTRACT

A blind frequency domain method for estimating the propagation delays of DS-CDMA signals is presented. The algorithm is formulated using an antenna array, and uses the fact that the spatially and temporally correlated additive noise, consisting of both interfering users and thermal noise, will be *asymptotically* uncorrelated on the FFT frequency grid. The algorithm is computationally simple, and can be efficiently implemented using the FFT algorithm. Simulations illustrate that near-far resistance can be achieved using spatial diversity.

1. INTRODUCTION

Direct-sequence code division multiple access (DS-CDMA) is widely considered to be a promising technology for future wireless communication networks. Due to the near-far problem for CDMA cellular systems in a multiuser environment, there has lately been an increased interest in robust time-delay estimation and blind adaptive interference suppression (see [1] and the many references therein). Near-far resistant estimators are of great importance in cases without closed loop power control schemes, as it is well known that the standard detector becomes useless in cases where the power received from different users becomes unequal.

This paper focuses on a novel method for *efficiently* computing an initial propagation delay estimate, to be used by, for instance, the Approximative Maximum Likelihood (AML) algorithm presented in [2]. The latter algorithm is known to perform poorly for the acquisition problem, as the AML cost function is highly nonlinear with many local minima, but performs well for the tracking problem. Similarly, exploiting an accurate initial timing estimate can significantly simplify several of the algorithms in the recent literature. One of the key aspects of the proposed technique is that it can be implemented using an arbitrary antenna array. It was shown in [3] that exploiting spatial diversity leads to substantial performance advantage for CDMA systems. Our simulations confirm this, and it is found that by the introduction of spatial diversity, the estimator becomes increasingly near-far resistant.

The presented algorithm only assumes knowledge of the spreading sequence of the desired user, and can be efficiently implemented using the FFT algorithm. The algorithm can easily be rewritten to handle the use of a known training sequence.

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2. PROBLEM FORMULATION

The paper considers an asynchronous K-user DS-CDMA system operating over an additive white Gaussian noise channel. The modulation is assumed to be binary phase shift keying (BPSK), although the algorithm may be applied to any type of PSK modulation. The transmitted signal for the k:th user, $s_k(t)$, is formed by spreading the k:th user's data stream, $g_k(m) \in \{+1, -1\}$, with the pulse shaping function, $h(t)$, i.e.,

$$s_k(t) = \sum_{m=-\infty}^{\infty} g_k(m) h(t - mT_c), \qquad (1)$$

where T_c is the chip period. The data stream, $g_k(m)$, is formed by modulating the k:th user's bit information, $b_k(n)$, with the k:th user's signature sequence $c_k(n)$:

$$g_k(m) = \sum_{n=0}^{N-1} b_k(n) c_k(m - nN), \tag{2}$$

where the processing gain N is the ratio of the bit and chip periods, $N \triangleq T_b/T_c$. The transmitter structure is illustrated in Figure 1. The pulse shaping modulation of the spread

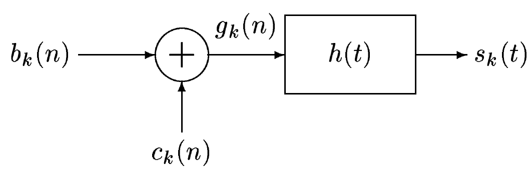

Figure 1: Transmitter structure

data sequence is not, as is commonly the case (see, e.g., [1, 2]), assumed to be a rectangular waveform, but is instead more realistically modeled as a known bandlimited signal (e.g., as in [4]). This bandwidth constraint is likely to be imposed on any commercial system.

We consider a case with short spreading sequences. This means that $c_k(n)$ is periodic with period equal to the processing gain N. In Section 6 we outline how the proposed algorithm can be extended to long, or random, codes.

We assume that the signals from the K users are received at an M-element antenna array in the presence of additive white Gaussian noise. It should be noted here that the use of an antenna array is not necessary for the formulation of the estimator. However, as we shall illustrate in Section 7, the introduction of an antenna array will make the estimator increasingly near-far resistant.

At each antenna element, the received signal is passed through a filter matched to the pulse shaping function and sampled at the sampling interval $T_i = T_c/Q$, where Q is an integer and referred to as the oversampling factor. The receiver structure is depicted in Figure 2. The sampled signal,

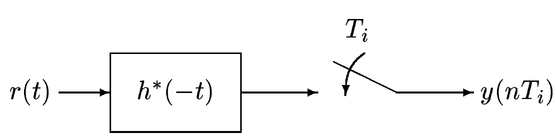

Figure 2: Receiver structure

$\mathbf{y}_t(nT_i)$, can then be written as

$$\mathbf{y}_t(nT_i) = \mathbf{a}_k s_k(nT_i - \tau_k) + \mathbf{e}_t(nT_i), \tag{3}$$

for $n = 0, \ldots, LNQ - 1$, where L is the number of transmitted bits, τ_k denotes the propagation delay, and $\mathbf{a}_k$ the $M \times 1$ spatial signature of the antenna array for user k. The additive noise sequence, $\mathbf{e}_t(n)$, is the temporally and spatially correlated interference consisting of both the $K - 1$ co-channel users and the additive thermal noise.

The standard narrowband assumption is employed here; i.e., the propagation time of the signal across the array is assumed to be much less than the reciprocal of the signal bandwidth. To simplify the problem, we do not use an explicit parameterization of the spatial response in terms of directions of arrival (DOA), but instead treat the elements of $\mathbf{a}_k$ as deterministic parameters to be estimated. This allows us to consider a cluster of coherent arrivals that share a given time delay, without the necessity of estimating the number of such arrivals nor their individual DOAs and amplitudes. In addition, this assumption eliminates the need for an accurately calibrated array.

3. PROPOSED ESTIMATOR

The proposed algorithm for estimating τ_k first converts the received signal into a sequence of vectors by dividing the signal into $L-1$ overlapping blocks of length $2NQ$ (for $n = 0, \ldots, 2NQ - 1$, and $j = 1, \ldots, L - 1$):

$$\mathbf{y}_t^j(n) = \mathbf{y}_t(nT_i + (j-1)NQT_i). \tag{4}$$

In this section, we derive the estimate of τ_k for a single j, and then show in the next section how to use the data in all L blocks. Let

$$p_t(t) = \sum_{m=0}^{N-1} c_k(m) h(t - mT_c) \tag{5}$$

denote the modulation of the spreading code for the k:th user. Each vector can then be seen as consisting of parts of three consecutive bits (see [1] for a discussion concerning this vectorization):

$$\begin{aligned} \mathbf{y}_t^j(n) &= \mathbf{a}_k b_k(j) p_t(nT_i - \tau_k) \\ &\quad + \mathbf{a}_k b_k(j-1) p_t(nT_i + T_b - \tau_k) \\ &\quad + \mathbf{a}_k b_k(j+1) p_t(nT_i - T_b - \tau_k) + \mathbf{e}_t^j(nT_i) \\ &= \mathbf{a}_k b_k(j) p_t(nT_i - \tau_k) + \mathbf{w}_t^j(nT_i) \end{aligned} \tag{6}$$

where $\mathbf{e}_t^j(nT_i)$ is the j:th noise vector and $\mathbf{w}_t^j(nT_i)$ denotes the additive noise combined with the "interfering" bits (i.e., the previous and the following bits, $j \pm 1$).

We then make use of the fact that the temporally correlated noise sequence, $\mathbf{w}_t^j(n)$, will be *asymptotically* (for large NQ) uncorrelated on the FFT frequency grid (see, e.g., [5]). Thus, using the time-shift property of the Fourier transform, we can approximately rewrite the received data in the frequency domain as

$$\mathbf{y}_\omega^j(l) = \mathbf{a}_k b_k(j) p_\omega(l) e^{-i\pi l \tau_k / NQ} + \mathbf{w}_\omega^j(l), \tag{7}$$

where $\tau_k = 0, \cdots, 2NQ-1$, and $p_\omega(l)$ and $\mathbf{w}_\omega^j(l)$ denote the l:th bin of the Fourier transform of $p_t(nT_i)$ and $\mathbf{w}_t^j(nT_i)$, respectively. According to [5], the additive noise $\mathbf{w}_\omega^j(l)$ in the frequency domain will be (asymptotically) uncorrelated in l, Gaussian distributed, with variance $\sigma_w^2(l)$, where

$$\sigma_w^2(l) = \left| \sum_{k=1}^{2NQ} \mathbf{w}_\omega^j(k) e^{-\pi i l k/NQ} \right|^2 . \tag{8}$$

In the following, $\sigma_w^2(l)$ is assumed to be known, and in the numerical examples is estimated as the averaged PSD of the M different sensors' measured signal vector. As the k:th users signal energy will only contribute with about one K:th the measured energy (and even less in unfavorable near-far situations), this approximation will be reasonably accurate.

We proceed with the derivation of the ML estimator for the j:th data vector, as written in (7), under the assumption that the additive noise in the frequency domain is Gaussian and spatially correlated, but uncorrelated in frequency.

Assuming the spatial statistics of the interference are frequency independent, we define the spatial noise covariance matrix, $\mathbf{Q}_w^l$, as

$$\begin{aligned} \mathbf{Q}_w^l &\triangleq E\left[\mathbf{w}_\omega^j(l)(\mathbf{w}_\omega^j(l))^*\right] \\ &= \sigma_w^2(l)\mathbf{Q}_w, \end{aligned} \tag{9}$$

where $(\cdot)^*$ denotes complex conjugate transpose. Let $\boldsymbol{\theta} = \{ \mathbf{Q}_w \quad \mathbf{a}_k \quad \tau_k \quad b_k(j) \}$ denote the unknown parameters. Also let $\tilde{\boldsymbol{\theta}} = \{ \mathbf{a}_k \quad \tau_k \quad b_k(j) \}$. The negative log-likelihood function for $2NQ$ samples of data is easily shown to be, after eliminating multiplicative and additive constants,

$$V_{2NQ}(\boldsymbol{\theta}) = \log|\mathbf{Q}_w^l| + \operatorname{tr}\left\{\mathbf{Q}_w^{-1}\mathbf{C}(\tilde{\boldsymbol{\theta}})\right\}, \tag{10}$$

where $|\cdot|$ and $\operatorname{tr}\{\cdot\}$ denote the determinant and the trace operation, respectively, and

$$\mathbf{C}(\tilde{\boldsymbol{\theta}}) = \frac{1}{2NQ} \sum_{l=1}^{2NQ} \mathbf{e}_\omega(l)\mathbf{e}_\omega^*(l)\sigma_w^{-2}(l) \tag{11}$$

$$\mathbf{e}_\omega(l) = \mathbf{y}_\omega^j(l) - \mathbf{a}_k b_k(j) p_\omega(l) e^{-i\pi l \tau_k/NQ}. \tag{12}$$

Using standard matrix calculus (see, e.g., [6]), the gradient of the criterion in (10) with respect to $\mathbf{Q}_w$ is easily shown to be

$$\frac{\partial V_{2NQ}(\boldsymbol{\theta})}{\partial \mathbf{Q}_w} = \mathbf{Q}_w^{-1} - \mathbf{Q}_w^{-1}\mathbf{C}(\tilde{\boldsymbol{\theta}})\mathbf{Q}_w^{-1} \tag{13}$$

from which it is clear that the ML estimate of $\mathbf{Q}_w$ is given by

$$\hat{\mathbf{Q}}_w = \mathbf{C}(\tilde{\boldsymbol{\theta}}). \tag{14}$$

When (14) is substituted into (10), the concentrated criterion

$$V_{2NQ}(\tilde{\boldsymbol{\theta}}) = \log|\mathbf{C}(\tilde{\boldsymbol{\theta}})| + M \tag{15}$$

results.

Dropping the scaling by $2NQ$, we can rewrite (11) as

$$\begin{aligned} \mathbf{C}(\tilde{\boldsymbol{\theta}}) &= \left(\mathbf{Y}_\omega^j \mathbf{V}_\tau - \mathbf{a}_k b_k(j)\mathbf{p}_\omega\right) \boldsymbol{\Lambda} \left(\cdot\right)^* && (16) \\ &= \left(\tilde{\mathbf{Y}}_\omega^j \mathbf{V}_\tau - \mathbf{a}_k b_k(j)\tilde{\mathbf{p}}_\omega\right) \left(\cdot\right)^* && (17) \end{aligned}$$

where $(\cdot)^*$ denotes the first parenthesis transpose conjugated,

$$\begin{aligned} \mathbf{v}_\tau &= \left[\begin{array}{ccc} e^{\pi i \tau_k/NQ} & \dots & e^{2\pi i \tau_k} \end{array}\right]^T \\ \mathbf{p}_\omega &= \left[\begin{array}{ccc} p_\omega(1) & \dots & p_\omega(2NQ) \end{array}\right] \\ \tilde{\mathbf{p}}_\omega &= \mathbf{p}_\omega \boldsymbol{\Lambda}^{1/2} \\ \mathbf{V}_\tau &= \operatorname{diag}(\mathbf{v}_\tau) \\ \mathbf{Y}_\omega^j &= \left[\begin{array}{ccc} \mathbf{y}_\omega^j(1) & \dots & \mathbf{y}_\omega^j(2NQ) \end{array}\right] \\ \tilde{\mathbf{Y}}_\omega^j &= \mathbf{Y}_\omega^j \boldsymbol{\Lambda}^{1/2} \\ \boldsymbol{\lambda} &= \left[\begin{array}{ccc} \sigma_w^{-2}(1) & \dots & \sigma_w^{-2}(2NQ) \end{array}\right] \\ \boldsymbol{\Lambda} &= \operatorname{diag}(\boldsymbol{\lambda}), \end{aligned}$$

$\operatorname{diag}(\mathbf{v}_\tau)$ denotes a matrix with the vector $\mathbf{v}_\tau$ along its diagonal, and $(\cdot)^T$ denotes the matrix transpose. Minimization of $V_{2NQ}(\tilde{\boldsymbol{\theta}})$ in (15) with respect to $\mathbf{a}_k$ yields

$$\hat{\mathbf{a}}_k = \frac{\tilde{\mathbf{Y}}_\omega^j \mathbf{V}_\tau \mathbf{p}_\omega^*}{\mathbf{p}_\omega \mathbf{p}_\omega^* b_k(j)}, \tag{18}$$

and inserting (18) in (17) gives

$$\mathbf{C}(\tau_k) = \mathbf{R}_j - \boldsymbol{\alpha}(\tau_k)\boldsymbol{\alpha}^*(\tau_k) \tag{19}$$

where

$$\begin{aligned} \mathbf{R}_j &= \mathbf{Y}_\omega^j \boldsymbol{\Lambda} \mathbf{Y}_\omega^{j*} \\ \boldsymbol{\alpha}(\tau_k) &= \frac{\tilde{\mathbf{Y}}_\omega^j \mathbf{V}_\tau \tilde{\mathbf{p}}_\omega^*}{\sqrt{\tilde{\mathbf{p}}_\omega \tilde{\mathbf{p}}_\omega^*}}. \end{aligned}$$

It is worth noting that the cost criterion in (19) only depends on the propagation delay, not on the transmitted bit. This is a result of the fact that $b_k(j)b_k^*(j) = 1$ in (19).

By making use of the determinant rules $|AB| = |A||B|$ and $|I + AB| = |I + BA|$ the concentrated criterion (15) can be expressed as

$$V_{2NQ}(\tau_k) = \log\left(\ 1 - \boldsymbol{\alpha}^*(\tau_k)\mathbf{R}_j^{-1}\boldsymbol{\alpha}(\tau_k)\ \right) \tag{20}$$

where terms not depending on τ_k have been dropped. Thus

$$\begin{aligned} \hat{\tau}_k &= \arg\max_{\tau_k} \boldsymbol{\alpha}^*(\tau_k)\mathbf{R}_j^{-1}\boldsymbol{\alpha}(\tau_k) \\ &= \arg\max_{\tau_k} \mathbf{v}_\tau^* \boldsymbol{\beta}_j \mathbf{R}_j^{-1} \boldsymbol{\beta}_j^* \mathbf{v}_\tau \\ &= \arg\max_{\tau_k} \|\mathbf{v}_\tau^* \boldsymbol{\beta}_j \mathbf{R}_j^{-1/2}\|^2, \end{aligned} \tag{21}$$

where $\|\cdot\|$ denotes the Euclidean norm, and

$$\begin{aligned}\boldsymbol{\beta}_j &= \mathrm{diag}(\mathbf{p}_\omega)\,\boldsymbol{\Lambda}\,\mathbf{Y}_\omega^{j*} \\ &= \left(\mathbf{e}_M\left(\mathbf{p}_\omega \odot \boldsymbol{\lambda}\right)\right)^T \odot \mathbf{Y}_\omega^{j*}, \end{aligned} \tag{22}$$

where $\mathbf{e}_M = \left[\begin{array}{ccc} 1 & \dots & 1 \end{array}\right]^T$ is $M \times 1$, and where $\odot$ denotes the Schur-Hadamard product.

Due to the presence of noise in (20), $\boldsymbol{\alpha}^*(\tau_k)\mathbf{R}_j^{-1}\boldsymbol{\alpha}(\tau_k) < 1$ with probability one and the criterion is well defined (see also [7]). Since $\mathbf{v}_\tau$ is a DFT vector, the maximum of the criterion in (21) is simply the maximum of the sum of M FFTs. Note that the computational complexity can be lowered significantly by evaluating $\mathbf{R}_j$ as

$$\mathbf{R}_j = \left(\mathbf{Y}_\omega^j \odot (\mathbf{e}_M \boldsymbol{\lambda})\right)\mathbf{Y}_\omega^{j*}. \tag{23}$$

It is worth stressing again that the cost function in (19) is *independent* of the transmitted bit. This can also be seen from (21) which does not depend on $b_k(j)$.

4. COMBINATION OF MULTIPLE BITS

Clearly it is preferable to consider all the $LNQ \times m$ samples in the estimation of τ_k at the same time. This is possible if we know all the transmitted bits, as is the case when training symbols are available. In this case, we form a block $\mathbf{y}_t^j(n)$ which contains the (spread) training sequence, and apply the proposed algorithm on that block. The signal $b_k(j)p_t(nT_i)$ will in this case span the entire training sequence (with $b_k(j)$ now denoting all the known training bits, and $p_t(nT_i)$ the corresponding modulated codewords).

Without knowledge of the transmitted signal, we propose an *ad-hoc* combining of the $L-1$ data vectors in (7) by adding their respective "pseudospectras", as formed in (21), prior to finding the maximum, i.e.,

$$\hat{\tau}_k = \arg\max_{\tau_k} \sum_{j=1}^{L-1} \left\| \mathbf{v}_\tau^* \boldsymbol{\beta}_j \mathbf{R}_j^{-1/2} \right\|^2. \tag{24}$$

5. COMPUTATIONAL COMPLEXITY

The algorithm is found to be computationally simple, requiring roughly $O(LMNQ\log[NQ])$ operations. This is to be compared with, for instance, the MUSIC algorithm proposed by Ström *et al.* [2] which requires about $O(L[NQ]^2 + [NQ]^3)$ operations. Typically, the first $L[NQ]^2$, which are due to the computation of the sample covariance matrix, will form a significant part of the computational load, as the MUSIC algorithm requires about $L = 100$ symbols to give an accurate estimate of the covariance matrix. See Section 7 for a study of how the number of symbols affect the estimation accuracy for the proposed algorithm.

6. EXTENSION TO LONG CODES

The main idea of the proposed algorithm is that the block $\mathbf{y}_t^j(n)$ defined in (4) must contain a signal, whose Fourier transform is known up to a complex constant. With short codes, this is easy to accomplish: any block of length $2T_b$ will contain a period of the code sequence. With long codes, this is not the case; the spreading code varies from symbol to symbol. Since we are not synchronized, we do not know which part of the signature sequence is used to spread the current symbol.

A solution to this problem is to increase the length of the block $\mathbf{y}_t^j(n)$, so that we are sure it contains a given part of the long spreading code. For instance, if we know that the signal from the transmitter arrives within a period of 10 symbols, we form blocks of length $10T_b$ and apply the algorithm described in Section 3. Since the blocks are overlapping, the number of blocks will only decrease slightly.

With this trick, the algorithm can be applied in a system employing long codes. The extension of the blocks will cause a higher noise level: the variance of the term $\mathbf{w}_t^j(nT_i)$ will increase. In practice, long codes are preferred due to the inherent interference averaging they provide, which is also advantageous for the proposed algorithm.

7. NUMERICAL RESULTS

To evaluate the proposed algorithm, Monte Carlo simulations were performed. The simulated system was a 10-user scenario with $N = 31$ chips per bit and $T_c = 1$ Gold code sequences. The measured data was sampled $Q = 3$ times per chip. The desired user's signal is assumed to be impinging on the array from broadside with a (randomly chosen) time-delay of $\tau_k = 3.951T_c$, whereas the other users have random DOAs and time-delays. The carrier power for the desired user is assumed to be one, $P_0 = 1$, while all the interfering users have the same received power, $P_k = P_1$, for $k = 1, \dots, K-1$. The Near-Far-Ratio (NFR) is defined as NFR = P_1/P_0. In the figures presented below, we illustrate the probability of a correct acquisition, i.e., the probability that the estimate is within a half sample of the true value. A total of 500 Monte Carlo runs were done for each simulation.

Figure 3 shows the estimator's code acquisition probability as the signal-to-noise ratio (SNR) and the number of transmitted symbols, L, varies. We see that with only $L = 10$ symbols, the propagation delay can be reliably estimated using three sensors in over 95% of the cases for SNR $\geq$ 0 dB. As seen in Figure 4, spatial diversity can be used to give near-far resistance. The figure shows the probability of a correct acquisition as the NFR, and the number of antennas, M, varies.

As was mentioned in section 4, the algorithm can be im-

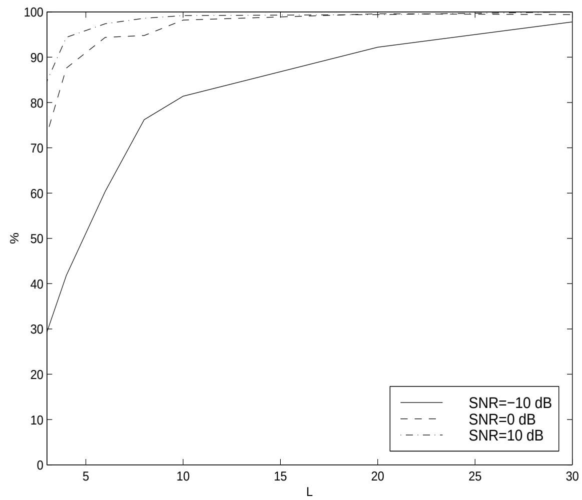

Figure 3: Probability of correct acquisition; NFR = 1, $M = 3$ sensors.

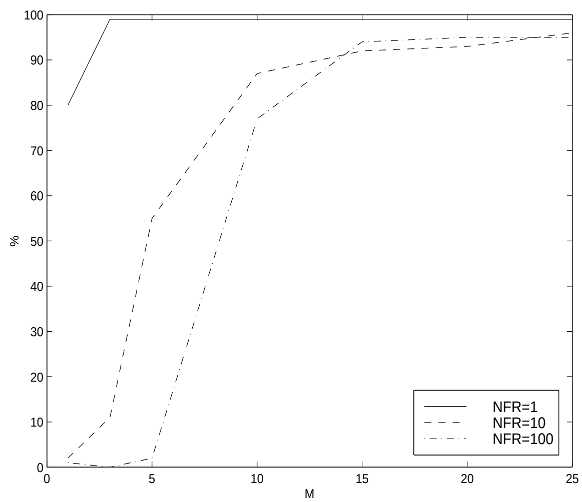

Figure 4: Probability of correct acquisition; $L = 20$ symbols, SNR = 10 dB.

plemented using training sequences. In the following example we examine the code acquisition probability for a 18 symbol training sequence. The symbol synchronization of the training sequence is assumed to be known, i.e., the k:th users time-delay is known within one symbol period.

Figure 5 shows the estimator's code acquisition probability as the NFR and the number of antennas varies. As can be seen by comparing Figure 4 and Figure 5, the use of the training sequence improves the estimator's performance, but interestingly not by much.

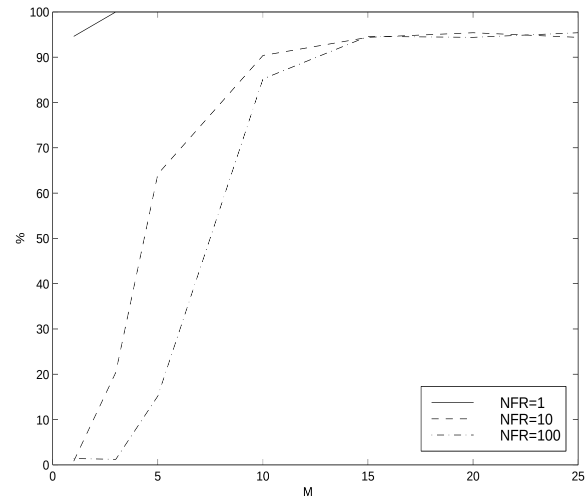

Figure 5: Probability of correct acquisition for a known 18 symbols long training sequence; $L = 20$ symbols, SNR = 10 dB.

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