Systematic Anti-Windup Compensator Design for Multivariable Systems*

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Abstract

The aim of anti-windup compensation is to modify the dynamics of a control loop when control signals saturate, so that a good transient behaviour is attained after desaturation, while avoiding nonlinear oscillations and repeated saturations.

Model-based anti-windup compensation is here considered for multiple-input multiple-output (MIMO) systems. A modified controller structure is proposed, which leaves the nominal closed-loop dynamics unchanged, as long as none of the control signals saturate. The proposed approach is applicable to continuous-time as well as discrete-time systems. Although it is developed for systems in input-output form, it can be used for systems in state-space form as well.

1 Introduction

The problem of finding controllers which have desired properties during or after saturation events has, over the years, resulted in a number of different anti-windup strategies. Many of the proposed methods focus on adjusting the states of the controller during a saturation event. For reasons explained in [6], and more recently also in [3, 4], there is, however, no guarantee that the *whole* system behaves acceptably during or after a saturation event, when only *controller*-state windup is prevented. Repeated saturations and even limit cycles might occur. To avoid such effects, the whole linear dynamics around the saturating elements, consisting of nominal controller elements, anti-windup filters and the open-loop plant, has to be taken into account. In the scalar case this can be accomplished in a Nyquist diagram: the loop gain around the saturating element is adjusted so that it stays well away from the function $-1/Y(C)$, where $Y(C)$ is the describing function of the saturation nonlinearity. The aim of the present paper is to introduce a controller structure which makes it possible to generalize this technique for analysis and design to feedback systems with multiple control signals. A key simplification is that the loop gain

*This work is supported by the NUTEK program on Industrial Control, REGINA. It is performed within the project Robust multivariable industrial control, in cooperation with ABB Industrial Systems.

relevant for the saturation behaviour is made diagonal. The diagonal elements can then be adjusted in the same way as for a scalar system.

2 Problem formulation

The aim of anti-windup design, as it will be presented here, is to obtain a good transient behaviour after desaturation. With a good transient behaviour we mean that

1. desaturation transients should have a fast decay;
2. limit cycles and repeated saturations should not occur.

To achieve these aims, a method developed in [8] will be utilized.

Consider a discrete-time[1] multivariable, stable or marginally stable system with m inputs and p outputs, parameterized in rational fractional form as

$$\begin{aligned} y(k) &= \boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)v(k) \\ v(k) &= sat[u_W(k)] \quad . \end{aligned} \tag{1}$$

Above, $\boldsymbol{\mathcal{A}}(q)$ is assumed to be a diagonal stable rational matrix, with diagonal elements $\mathcal{A}_j$. Introduce the controller

$$u_W(k) = (\mathbf{I} - \boldsymbol{\mathcal{W}}(q)\boldsymbol{\mathcal{R}}(q))v(k) - \boldsymbol{\mathcal{W}}(q)\boldsymbol{\mathcal{S}}(q)y(k) + \boldsymbol{\mathcal{W}}(q)\boldsymbol{\mathcal{T}}(q)r(k) \tag{2}$$

where $\boldsymbol{\mathcal{W}}$, $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$, $\boldsymbol{\mathcal{T}}$ are stable rational matrices in q, of appropriate dimension. Here, $\boldsymbol{\mathcal{W}}$ is the anti-windup filter. The controller structure proposed in (2) is inspired by a similar structure suggested in [3] for scalar systems. It is depicted in Figure 1.

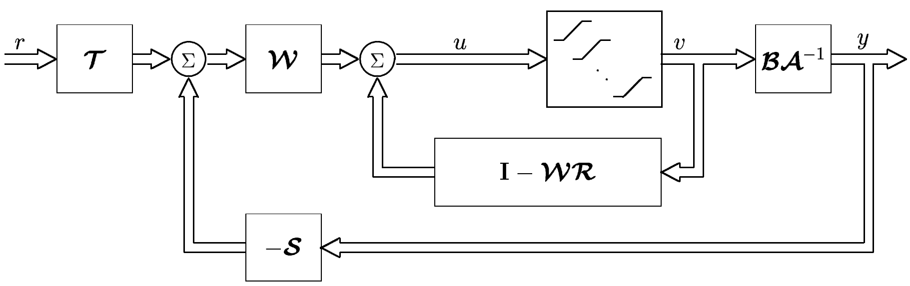


Figure 1: A discrete-time MIMO process $y(k) = \boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)\text{sat}[u(k)]$ with a two degree of freedom controller structure $\{\boldsymbol{\mathcal{R}}(q)\ \boldsymbol{\mathcal{S}}(q)\ \boldsymbol{\mathcal{T}}(q)\}$ appended with a stable and proper anti-windup transfer-operator matrix $\boldsymbol{\mathcal{W}}(q)$. The rational matrix $\boldsymbol{\mathcal{W}}(q)$ is to be selected such that the loop gain around the saturations becomes diagonal.

Remark: In the scalar case, the controller structure (2) includes a number of well know anti-windup schemes, of which the following are worth mentioning. (The matrices $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$ and $\boldsymbol{\mathcal{T}}$ are here assumed to be scalar polynomials R, S and T.)

[1] The anti-windup concept presented here is applicable to both discrete time and continuous time systems. Here we shall however use a discrete time framework, based on the forward shift operator q, $(qy(k) = y(k+1))$.

1. *The observer-based method of Åström and Wittenmark* [7] is obtained if $\boldsymbol{\mathcal{W}} = F^{-1}$, where F is the characteristic polynomial of the anti-windup observer.

2. *The conditioning technique of Hanus* [2] is obtained if $\boldsymbol{\mathcal{W}} = t_0 T^{-1}$, where t_0 is the leading element of T.

For more details see [4, 5].

Following [4], let us regard the difference between the actual and the saturated control signal as an exogenous disturbance

$$\delta(k) \triangleq v(k) - u_W(k) \quad . \tag{3}$$

By omitting the argument q, and combining (1)-(3), the closed-loop system is then obtained as

$$y(k) = y_{nom}(k) + y_\delta(k) = \boldsymbol{\mathcal{H}}_{nom} r(k) + \boldsymbol{\mathcal{H}}_\delta \delta(k) \tag{4}$$

where

$$\boldsymbol{\mathcal{H}}_{nom} = \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{T}} \quad ; \quad \boldsymbol{\mathcal{H}}_\delta = \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{W}}^{-1} \quad . \tag{5}$$

Above $\boldsymbol{\mathcal{H}}_{nom}$ constitutes the nominal closed-loop system which is obtained in (4) when $\delta = 0$, i.e. when the control signals do not saturate. When a control signal exits saturation, $\boldsymbol{\mathcal{H}}_\delta$ will determine the resulting transient behaviour. According to the specified requirements above, the dynamics of $\boldsymbol{\mathcal{H}}_\delta$ should be fast. This can be achieved by appropriate choices of $\boldsymbol{\mathcal{W}}$. However, if the dynamics of $\boldsymbol{\mathcal{H}}_\delta$ is made too fast, then repeated saturations and limit cycles may occur. Thus, the requirements 1. and 2. above are often contradictory. It is therefore essential that an anti-windup design includes a trade-off between a fast transient and a small influence of nonlinear effects. A design method is presented next, which utilizes simple scalar tools for attaining such a trade-off .

3 Systematic anti-windup design

For the design of the anti-windup filter $\boldsymbol{\mathcal{W}}$ in (2), the criterion

$$J = \|\boldsymbol{\mathcal{H}}_\delta\|_2^2 + \left\|\mathbf{Q}\left((\boldsymbol{\mathcal{L}}_v + \mathbf{I})^{-1} - \mathbf{I}\right)\right\|_2^2 \tag{6}$$

is introduced. This criterion is a generalization of a criterion suggested for the scalar case by Sternad and Rönnbäck in [5]. In (6), $\mathbf{Q}$ is a diagonal penalty matrix whereas $\boldsymbol{\mathcal{L}}_v$ represents the loop gain around the saturation nonlinearity. Now, by choosing $\boldsymbol{\mathcal{W}}$ as

$$\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1} \quad , \tag{7}$$

where $\boldsymbol{\mathcal{P}}$ is a stable and rational matrix to be determined, the rational matrices $\boldsymbol{\mathcal{H}}_\delta$ and $\boldsymbol{\mathcal{L}}_v$ are given by

$$\boldsymbol{\mathcal{H}}_\delta = \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{P}}^{-1} \; ; \; \boldsymbol{\mathcal{L}}_v = \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}^{-1} - \mathbf{I} \tag{8}$$

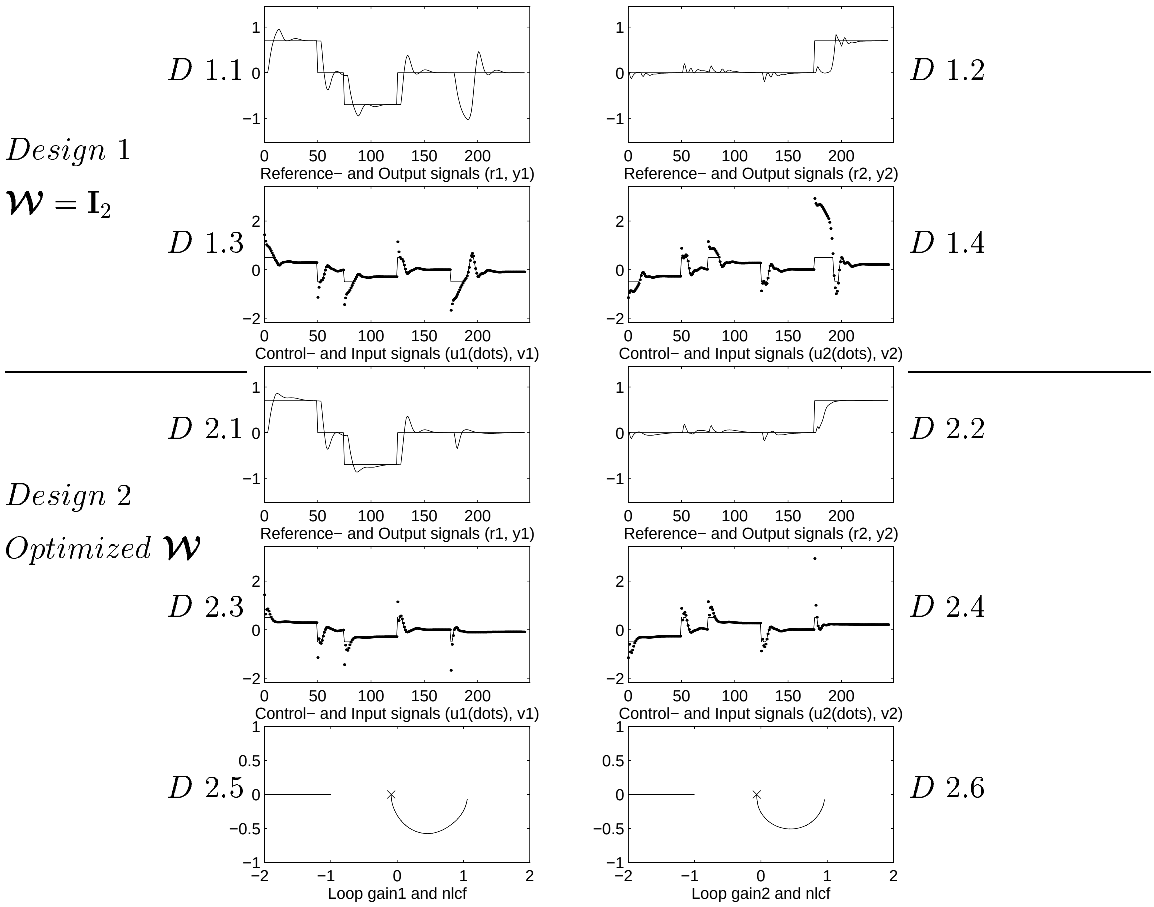
respectively. If $\boldsymbol{\mathcal{P}}$ is choosen *diagonal*, then $\boldsymbol{\mathcal{L}}_v$ will be diagonal. By insertion of (8) into (6), the criterion can be rewritten as

$$J \; = \; \left\|\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{P}}^{-1}\right\|_2^2 + \left\|\mathbf{Q}\left(\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{P}}^{-1} - \mathbf{I}\right)\right\|_2^2 \quad . \tag{9}$$

Minimizing (9), with respect to $\boldsymbol{\mathcal{P}}$, for a given penalty matrix $\mathbf{Q}$, is shown in [8] to be equivalent to the solution of m separate scalar spectral factorization equations

$$r_j \mathcal{P}_j \mathcal{P}_j^* \; = \; \sum_{i=1}^{p} \mathcal{B}_{ij}\mathcal{B}_{ij}^* + \rho_j \mathcal{A}_j \mathcal{A}_j^* \; ; \; \boldsymbol{\mathcal{P}} = \mathrm{diag}(\mathcal{P}_j) \quad . \tag{10}$$

Here, (10) has to be solved for $j = 1, 2....m$, where m is the number of process inputs, p is the number of process outputs, r_j is a scale factor and ρ_j is the jth diagonal element of $\mathbf{Q}$. The design of a multivariable anti-windup compensator is thus reduced to m scalar designs, in which the m elements of the diagonal loop gain matrix $\boldsymbol{\mathcal{L}}_v$ are systematicaly adjusted. Note that if $\rho_j \to \infty$, then $\mathcal{P}_j \to \mathcal{A}_j$. The jth loop gain $\mathcal{L}_j$ will then contract and stay well away from the negative real axis. As a result, repeated saturations will not occur in that loop. However, the desaturation transients may then show an unsatisfactory behaviour, since the common denominator of the jth column of $\boldsymbol{\mathcal{H}}_\delta$ goes towards the plant dynamics $\mathcal{A}_j$. On the other hand if ρ_j is selected small, the dynamics of the jth column of $\boldsymbol{\mathcal{H}}_\delta$ will become fast, while the jth loop gain may become large. This, in turn, may generate repeated saturations and limit cycles. The user must therefore select the values of ρ_j properly to obtain an appropriate trade-off.



4 Example

The controller in (2) is used for two different choices of $\boldsymbol{\mathcal{W}}$. In both the cases, the nominal controller, $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$, $\boldsymbol{\mathcal{T}}$, originates from an observer-based state-feedback LQ-control law, expressed in input-output form. The input penalty is 0.01 for both inputs. The model used for simulation descibes a *Heavy Oil Fractionator* [1], with two inputs and two outputs. The model used for controller- and anti-windup filter design, is obtained by subspace-identification. In the first example, we select $\boldsymbol{\mathcal{W}} = \mathbf{I}$, which simply means that the observer is fed with saturated control signals. The result of the simulation is shown in the four upper diagrams, (D 1.1 – D 1.4). In the other example, the method proposed in this paper was used for the design of $\boldsymbol{\mathcal{W}}$. The result, after adjustment of the penalties ρ_1, ρ_2, is shown in the six lower diagrams, (D 2.1 – D 2.6). The two bottom diagrams show the diagonal elements of the loop gain $\mathcal{L}_v$ and the functions $-1/Y(C)$.

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