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# Making SMI-Beamforming Insensitive to the Sampling Timing for GSM Signals

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ABSTRACT Beamforming with the Sample Matrix Inversion method (SMI), using an antenna array, is considered for a GSM-signal. The problem of generating the reference signal from the training sequence is addressed. As the proper reference signal to be used varies with the sampling timing within the symbol interval, a modified SMI method is proposed that uses a "variable" reference signal. When working with a short training sequence, as in GSM, the performance of the standard SMI beamforming method is shown to degrade when there is an offset from the sampling instant that the reference signal was designed for. The proposed modified SMI method, however, is shown to be more or less insensitive to where in the symbol interval the sampling takes place. The proposed method achieves a better mean performance, averaged uniformly over one symbol interval.

## 1. Introduction

In the European digital mobile cellular phone system, GSM, Gaussian Minimum Shift Keying (GMSK) is used for the modulation. The resulting digital channel from transmitted symbols to received samples can approximately be described by an FIR filter. However, depending on the sampling timing, this channel varies.

If an antenna array is utilized, beamforming can be performed by adjusting multiplicative weights, assigned to the different antenna elements, in order to make the resulting signal as similar as possible to the reference signal. This can for example be performed with the Sample Matrix Inversion method (SMI). See [1].

In order to use the SMI method, a reference signal is required. As there is a training sequence in each data burst, this can be used to create a reference signal. If the discrete time channel, approximating the channel between the transmitted symbols and received samples, was independent of the sampling timing, we could form an "exact" reference signal by filtering the training sequence with a fixed filter. Generation of a reference signal with a fixed filter is for instance used in [2]. However, the channel varies with the sampling timing. Let us assume that the sampling instant is not synchronized within the symbol interval, implying that the location of the sampling instant within the symbol interval is unknown. In this case the modulation and sampling can be approximated by a set of discrete time channels, parametrized by the location of the sampling instant within the symbol interval.

If one of these channels is selected in order to create the reference signal and the true channel is a different one, then there will be a discrepancy between the reference signal selected for the antenna array to receive optimally, and the actual samples received. When working with short training sequences, this can cause a degradation in the signal to interference and noise ratio (SINR) after the beamformer.

A modified weight adaptation scheme that circumvents this problem is proposed here. The idea is to introduce some degrees of freedom in the reference signal.

## 2. Approximating the GSM Modulation and Sampling with Discrete Time Channels

A model of the GSM channel from transmitted symbols to received samples can be seen in Figure 1. Differential encoding is performed by the operation $\ominus$, here defined as: $1 \ominus 1 = 1$, $1 \ominus -1 = -1$, $-1 \ominus 1 = -1$ and $-1 \ominus -1 = 1$. The differential encoding is followed by GMSK modulation with a BT product of 0.3. The modulation bit rate is 270833 kbit/s

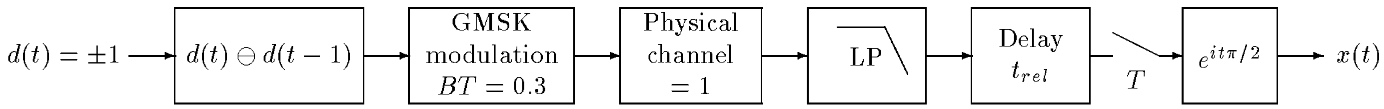


**Figure 1** *Model of the channel between the transmitted symbols and the received samples.*

| $B_{t_{rel}}(q^{-1})$ coefficients | | | | | |
|---|---|---|---|---|---|
| $t_{rel}$ $[T]$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
| $-0.5$ | $0.01\angle{-1.97}$ | $0.15\angle{+1.53}$ | $0.81\angle{-0.01}$ | $0.84\angle{-1.57}$ | $0.07\angle{-3.05}$ |
| $-0.4$ | $0.01\angle{-2.24}$ | $0.20\angle{+1.53}$ | $0.87\angle{-0.01}$ | $0.77\angle{-1.57}$ | $0.02\angle{-2.88}$ |
| $-0.3$ | $0.01\angle{-2.62}$ | $0.25\angle{+1.54}$ | $0.92\angle{-0.01}$ | $0.69\angle{-1.57}$ | $0.02\angle{-0.26}$ |
| $-0.2$ | $0.01\angle{-2.92}$ | $0.31\angle{+1.54}$ | $0.96\angle{+0.00}$ | $0.61\angle{-1.57}$ | $0.05\angle{-0.08}$ |
| $-0.1$ | $0.02\angle{-3.08}$ | $0.37\angle{+1.55}$ | $0.99\angle{+0.00}$ | $0.52\angle{-1.56}$ | $0.08\angle{-0.05}$ |
| $0.0$ | $0.03\angle{+3.12}$ | $0.44\angle{+1.55}$ | $1.00\angle{+0.00}$ | $0.44\angle{-1.56}$ | $0.10\angle{-0.03}$ |
| $0.1$ | $0.04\angle{+3.09}$ | $0.52\angle{+1.55}$ | $1.00\angle{+0.00}$ | $0.35\angle{-1.56}$ | $0.11\angle{-0.01}$ |
| $0.2$ | $0.06\angle{+3.09}$ | $0.59\angle{+1.56}$ | $0.98\angle{+0.00}$ | $0.28\angle{-1.55}$ | $0.11\angle{+0.00}$ |
| $0.3$ | $0.08\angle{+3.09}$ | $0.67\angle{+1.56}$ | $0.95\angle{+0.00}$ | $0.20\angle{-1.54}$ | $0.11\angle{+0.01}$ |
| $0.4$ | $0.11\angle{+3.09}$ | $0.74\angle{+1.56}$ | $0.90\angle{+0.01}$ | $0.13\angle{-1.53}$ | $0.10\angle{+0.03}$ |
| $0.5$ | $0.15\angle{+3.10}$ | $0.81\angle{+1.56}$ | $0.84\angle{+0.01}$ | $0.07\angle{-1.50}$ | $0.09\angle{+0.04}$ |

**Table 1** *Discrete time channels approximating the channel between the transmitted symbols and the received samples portrayed in Figure 1. The relative sampling instant, $t_{rel}$, is in units of a symbol interval. The channels are $B_{t_{rel}}(q^{-1}) = b_0 + b_1q^{-1} + b_2q^{-2} + b_3q^{-3} + b_4q^{-4}$, $(q^{-1}d(t) = d(t-1))$.*

($T = 3.69\mu$s). A detailed description of the GSM modulation can be found in [3]. Multipath propagation is not included here and the physical radio channel is set to unity. See comment in conclusion. The filters in the receiver is for simplicity modeled with a forth order Butterworth lowpass filter with a bandwidth of 90 kHz. The sampling timing is modeled with the delay $t_{rel}$. The sampling timing is thought to be unknown but constant during any single burst. After the sampling the samples are rotated with $\pi/2$ radians per sample.

Apart from a pure constant delay and a phase shift, the above described channel, can be approximated with the discrete time FIR filters, $B_{t_{rel}}(q^{-1})$, shown in Table 1 for different relative sampling instants, $t_{rel}$.

## 3. Sample Matrix Inversion with fixed reference signal filter

The reference signal for this algorithm is generated by filtering the training sequence of the burst through the filter with $t_{rel} = 0.0$, $B_{0.0}(q^{-1})$, in Table 1. The weights in the beamformer are then adjusted in order to make the received signal as close as possible to the the reference signal, in a MSE sense, as in the standard SMI-method. See [1] and Figure 2.

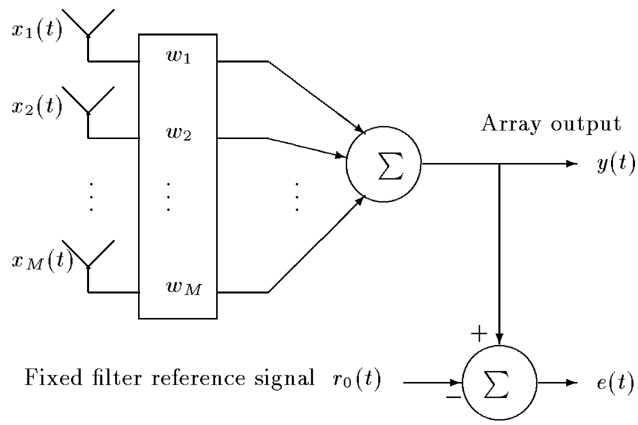


**Figure 2** *Configuration for the Sample Matrix Inversion Method with fixed reference signal filter.*

Define the vector $w$, consisting of the beamformer weights

$$w = [w_1\, w_2 \ldots w_M]^T \tag{1}$$

and the vector $x(t)$, consisting of the received signals from the antenna elements, after sampling,

$$x(t) = [x_1(t)\, x_2(t) \ldots x_M(t)]^T \tag{2}$$

The transmitted binary training symbols are denoted $d(t)$, $t$=1,2,...,$N$. The number of antenna elements is $M$.

The beamformer weights, $w$, are adjusted in order to minimize

$$V(w) = \sum_{t=5}^{N} |e(t)|^2 = \sum_{t=5}^{N} |y(t) - r_0(t)|^2 \qquad (3)$$

where $N$ is the number of symbols in the training sequence and $r_0(t)$ is the reference signal computed as

$$\begin{aligned} r_0(t) &= B_{0.0}(q^{-1})d(t) = b_0 d(t) + b_1 d(t-1) + \\ & \quad b_2 d(t-2) + b_3 d(t-3) + b_4 d(t-4) \end{aligned} \qquad (4)$$

and y(t) is the beamformer output

$$y(t) = w^T x(t) \qquad (5)$$

The filter coefficients $b_i$, i=0,1,..,4, in the generation of the reference signal correspond to the ones in Table 1 for $t_{rel} = 0.0$.

The minimum of (3) is approximately attained by the parameter vector

$$\hat{w} = (\hat{R}_{xx}^{-1} \hat{R}_{xr})^* \qquad (6)$$

where $(\cdot)^*$ represents elementwise complex conjugation. The matrices, $\hat{R}_{xx}$ and $\hat{R}_{xr}$, are sample covariance matrices computed from the training sequence data as

$$\hat{R}_{xx} = \frac{1}{N-4} \sum_{t=5}^{N} x(t) x^H(t) \qquad (7)$$

$$\hat{R}_{xr} = \frac{1}{N-4} \sum_{t=5}^{N} x(t) r^H(t) \qquad (8)$$

## 4. Sample Matrix Inversion with variable reference signal filter

In this proposed algorithm, the fixed part of the reference signal filter is the same as for the previous algorithm. Two variable components are however added to the reference signal. The idea is to improve the ability to lock on to a received signal sampled at a relative sampling instant different from zero, for instance at $t_{rel} = \pm 0.5$. The variable components added, $r_1(t)$ and $r_2(t)$, are the training sequence filtered through the difference between the channels at $t_{rel} = 0.0$ and $t_{rel} = \pm 0.5$, i.e.

$$r_1(t) = (B_{-0.5}(q^{-1}) - B_{0.0}(q^{-1}))d(t) \qquad (9)$$

$$r_2(t) = (B_{0.5}(q^{-1}) - B_{0.0}(q^{-1}))d(t) \qquad (10)$$

Other alternatives can also be considered. The general idea is to allow for some degrees of freedom in the reference signal. See Figure 3.

In addition to the weights in the adaptive beamformer, also the two coefficients, $c_1$ and $c_2$, for the variable components in the reference signal are adjusted, in order to minimize the error between the array output and the reference signal.

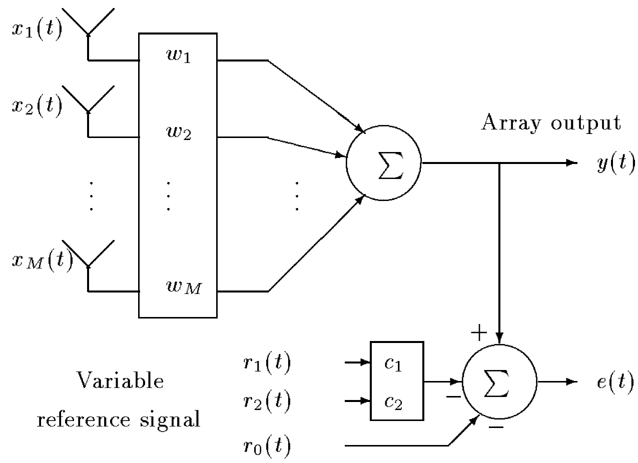


**Figure 3** *Configuration for the Sample Matrix Inversion Method with variable reference signal filter.*

Introduce the modified parameter and data vectors, $\theta$ and $z(t)$

$$\theta = [w_1 \, w_2 \ldots w_M \, c_1 \, c_2]^T \qquad (11)$$

$$z(t) = [x_1(t) \, x_2(t) \ldots x_M(t) \, r_1(t) \, r_2(t)]^T \qquad (12)$$

The modified parameter vector, $\theta$, is adjusted in order to minimize

$$V(\theta) = \sum_{t=5}^{N} |e(t)|^2 = \sum_{t=5}^{N} |\theta^T z(t) - r_0(t)|^2 \qquad (13)$$

where $r_0(t)$ is the reference signal computed as

$$r_0(t) = B_{0.0}(q^{-1})d(t) \qquad (14)$$

The minimum of (13) is approximately attained by the parameter vector

$$\hat{\theta} = (\hat{R}_{zz}^{-1} \hat{R}_{zr})^* \qquad (15)$$

The matrices $\hat{R}_{zz}$ and $\hat{R}_{zr}$ are computed as

$$\hat{R}_{zz} = \frac{1}{N-4} \sum_{t=5}^{N} z(t) z^H(t) \qquad (16)$$

$$\hat{R}_{zr} = \frac{1}{N-4} \sum_{t=5}^{N} z(t) r_0(t) \qquad (17)$$

## 5. Performance Evaluation

In order to evaluate the algorithms, the resulting SINR in the signals after the beamformers was computed. The algorithms were also evaluated by applying a maximum likelihood sequence estimator (MLSE) after the beamformer, and computing the resulting bit error rate (BER).

The MLSE uses an estimate of the channel that is formed by identifying the channel between the transmitted symbols and the received samples, after the beamformer.

## 6. Simulation Settings

A circular antenna array consisting of eight antenna elements, as shown in Figure 4, were used in the simulations. In other words $M$=8.

The desired signal is impinging on the array from the direction $\alpha = 0$ degrees, see Figure 5. Co-channel interferers are impinging on the array from the directions $\alpha_{co}$ = 135, -30 and -125 degrees, all having a constant channel $B_{co}(q^{-1}) = b_{co}$. The constant $b_{co}$ was selected such that the SIR, averaged over all the antenna elements, became 0 dB. Independent white noise giving a SNR of 3 dB, averaged over the antenna elements, was also added.

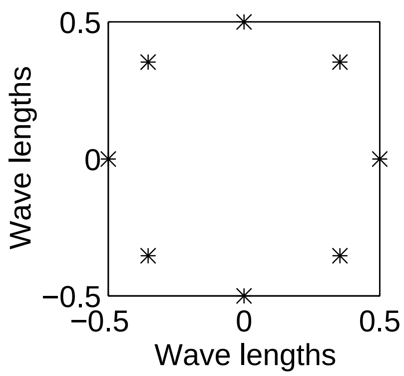


**Figure 4** *Antenna configuration*

The relative sampling instant, $t_{rel}$, was varied, resulting in different channels for the desired signal as in Table 1. The beamformers were tuned with data from a training sequence of 26 binary symbols, as in the GSM system. The channel used in the MLSE, was identified as a three tap FIR-channel, using the same training data. The SINR and BER was evaluated over 500 and 5000 symbols respectively. This experiment was repeated 100 times for different realizations of both the noise and interference.

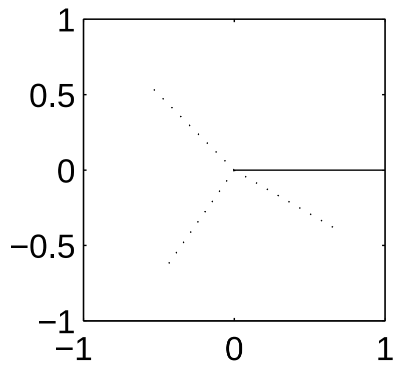


**Figure 5** *Desired signal (solid) and co-channel interferers (dotted).*

## 7. Results

In Figure 6, the SINR after the beamformers and the BER for the MLSE, can be seen as a function of the relative sampling instant $t_{rel}$. From the two diagrams it can be seen that the SMI beamformer with fixed reference signal filter has performance degradations when the relative sampling instant differs from $t_{rel} = 0.0$, which it was designed for. The SMI beamformer with variable reference signal filter is however more or less insensitive to the sampling timing within the symbol interval. Averaging uniformly over one symbol interval, gives better mean SINR and BER for the SMI method with the variable reference signal filter, see dotted lines in in Figure 6. However, if the deviation from $t_{rel} = 0.0$ is small, then the SMI method with fixed reference signal filter performs better.

The reason why the beamformer with the fixed reference signal filter has a performance degradation when $t_{rel}$ differs from zero, is that the incorrectly modeled channel causes the algorithm to treat part of the desired signal as a disturbance. The algorithm thus decreases the gain slightly in the direction of the desired signal, resulting in a lower SINR.

It should be noted that for longer training sequences the difference between the algorithms are not that pronounced. The results are connected with the use of a single short training sequence.

The results also depends on the signal to noise ratio. Adding more noise will make the difference between the algorithms smaller. The discrepancy in the reference signal will then be masked by the noise.

## 8. Conclusion

Beamforming with the SMI method working with the training data in a GSM burst has been studied. A scenario has been considered were the sampling instant is not synchronized within the symbol interval. A problem is then what reference signal to use. If a reference signal is used, formed by filtering the training sequence through a fixed FIR filter approximating the modulation, receiver filter and sampling, then the performance for the beamformer can degrade if the sampling instance differs from the one that the reference signal was designed for.

The proposed modified SMI method introduces some degrees of freedom in the reference signal. The algorithm then becomes more or less insensitive to the location of the sampling instant within the symbol interval. This results, for the scenario studied, in a lower mean BER, averaged uniformly over one symbol interval.

Multipath propagation has not been included in this study. It is believed that in the presence of multipath propagation, the received signal will differ even more from any reference signal generated from a fixed filter. Introducing some freedom in the reference signal should therefore be beneficial also in this case. The exact construction of the degrees of freedom may however be different from the construction presented here.

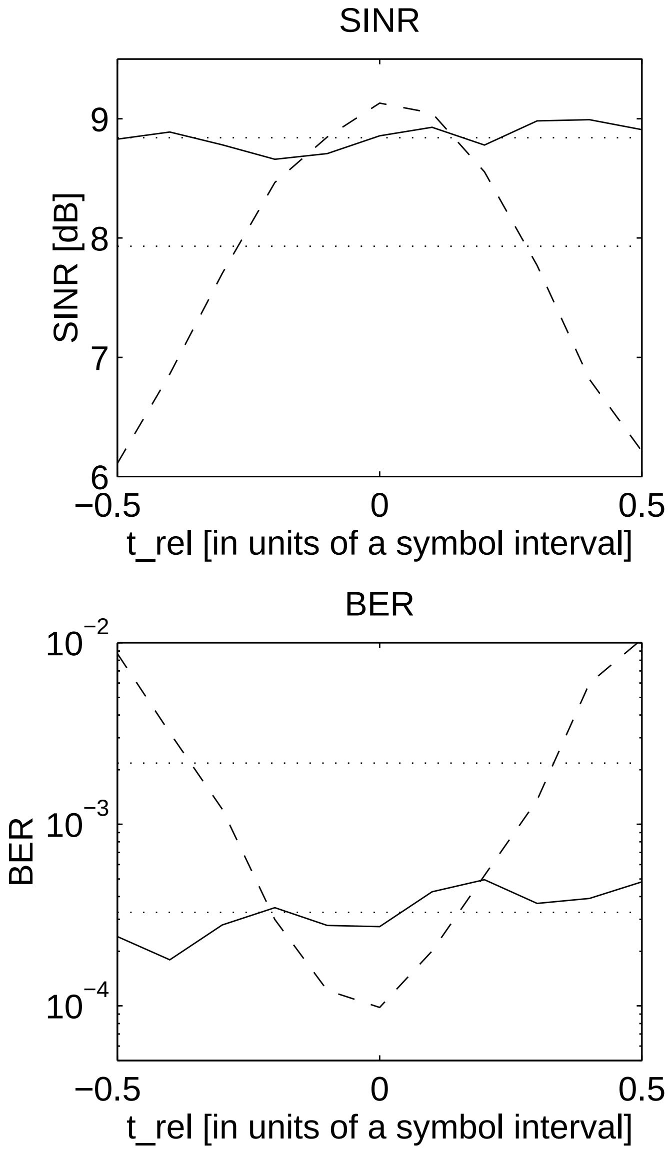


**Figure 6** *SINR after the beamformers and BER for the MLSE as a function of the relative sampling instant, $t_{rel}$. SMI with variable reference signal filter (solid) and SMI with fixed reference signal filter (dashed). The dotted lines are the respective mean SINR:S and BER:s, averaged uniformly over one symbol interval.*

## References

[1] J. R.T. Compton, *Adaptive Antennas.* New Jersey: Prentice-Hall, Inc, 1988.

[2] Y. Ogawa, Y. Nagashima and K. Itoh, “An adaptive antenna system for high-speed digital mobile communications,” *IEEE Transactions on Communications*, vol. E75-B, no. 5, pp. 413–21, May 1992.

[3] ETSI/PT 12, *GSM Technical Specification 05.04: Modulation*, Version 3.1.2.