ROBUST AND ADAPTIVE METHODS FOR EQUALIZATION, TRACKING AND DIVERSITY

Anders Ahlén, Mikael Sternad,
Lars Lindbom, Erik Lindskog, and Claes Tidestav.
Systems and Control Group, Uppsala University
P.O. Box 27, S–751 03 Uppsala, Sweden

ABSTRACT

Channel time variations and system capacity are issues that will persist in the design of future communication systems. In this paper we will focus on three different aspects to handle such problems: tracking, robustness and diversity. Rapidly time varying channels require new tracking methods, which are able to follow large phase jumps. When variations are moderate, robust methods could be employed to improve the average bit error rate. In order to improve capacity of existing systems, the use of multivariable methods is suggested.

1. INTRODUCTION

Information technology is one of todays fastes growing markets. The use of mobile phones will increase drastically during the remaining part of the nineties. Huge investments are also made in so called "electronical highways". However, it remains to connect subscribers to these highways. How this should be done has recently been actively debated. Suggestions include use of existing wires in conjunction with advanced signal processing. Another alternative is radio transmission from the gateways to the subscribers.

The requirement for new services will affect also the mobile network system specifications. New systems will use higher carrier frequency and cell utilization in urban areas will be further optimized, to increase capacity. We foresee two different scenarios: new systems will use higher carrier frequency, and smaller cells, whereas existing systems will have to be improved further to reach new quality and capacity goals. For both of these scenarios, as well as for the subscriber lines, we have to cope with time variable channel conditions and capacity issues. Sophisticated algorithms have to be developed to optimize throughput. This motivates further study and development of robust and adaptive methods for equalization, tracking and diversity.

The paper is organized in the following way. In Section 2 we discuss the use of robust decision feedback equalizers, as a means to improve average bit error rates in systems with relatively short symbol times. An example is included for a GSM scenario. Section 3 discusses new approaches to tracking of fast time varying parameters with special emphasis on mobile radio channels. In Section 4 we investigate the utility of multivariable equalizers as a means to improve capacity in existing systems. An example is included for the uplink case.

2. DESIGN OF ROBUST DECISION FEEDBACK EQUALIZERS

If data sequences $\{d_t\}$ are transmitted in the presence of intersymbol interference, they have to be reconstructed from the received sequences $\{y_t\}$. Equalizers compute estimates $\hat{d}_t$ on a symbol by symbol basis. Their main advantage, as compared to the MLSE Viterbi detector, is a low computational complexity.

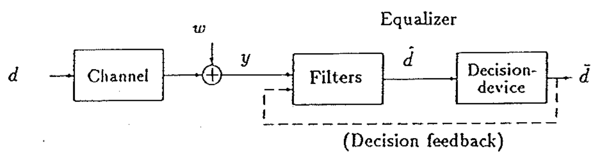

Figure 1: Decision feedback equalizer

Due to reflections from surrounding objects, the reception for a mobile radio varies as its location changes. This causes time variations (fading) in the coefficients of the channel.

If the time variations are large, equalizers that adapt to the channel have to be used. If on the other hand time variations are small, the filter coefficients can be adjusted during known training sequences, and held fixed until the next training. However, if in the latter case the time variations are not negligible, they have to be considered when designing the equalizer. This is an issue for a robust design algorithm.

Consider the following received, discrete-time, complex baseband signal

$$y_t = \left(B_0(q^{-1}) + \Delta B(q^{-1})\right) d_t + v_t \qquad (1)$$

where q^{-1} is the delay operator such that $q^{-1}y_t = y_{t-1}$. The transmitted symbols $\{d_t\}$ are assumed to be zero mean and white, with $E|d_t|^2 = \sigma_d^2$ whereas the noise v_t is zero mean, with variance $E|v_t|^2 = \sigma_v^2$. The nominal model of the transmission channel is described by a FIR filter with complex coefficients

$$B_0(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{nb} q^{-nb} \quad . \qquad (2)$$

This model is obtained eg by identification during the training sequence. Due to limited data records and noise, uncertainy is inevitable. The set of possible model errors is represented by the "error model"

$$\Delta B(q^{-1}) = \Delta b_0 + \Delta b_1 q^{-1} + ... + \Delta b_{nb} q^{-nb} \qquad (3)$$

where the coefficients Δb_i are stochastic variables with zero mean and known correlations $E[\Delta b_i \Delta b_j^*]$. The channel model above is a special case of more general uncertain models discussed in [4] and [6].

We now introduce the following IIR decision feedback equalizer (DFE)

$$\hat{d}_{t|t+\ell} = \frac{S(q^{-1})}{R(q^{-1})} y_{t+\ell} - \frac{Q(q^{-1})}{P(q^{-1})} \bar{d}_{t-1} \qquad (4)$$

where ℓ is a user-chosen smoothing lag and $\bar{d}_t$ represent decisioned data. The denominator polynomials $R(q^{-1})$ and $P(q^{-1})$ are required to be monic and stable. The optimization of such equalizers, based on exactly known channel and noise models, has been discussed in eg [2], [3]. The utility of using a robust equalizer is exemplified next.

Example

Consider a GSM–channel with three received signals having the average relative powers 0 dB, -1.8 dB and -4.8 dB, subject to independent Rayleigh fading. The three rays are received with a time separation of one symbol interval. Assume the carrier frequency to be 1800 MHz. The GSM-system uses a partial response modulation stretching over 3-4 symbol intervals. This results in a channel with 5-6 coefficients b_i. Fading due to movement of the mobile causes the channel to be slowly time varying over the duration of a burst. An example of this time variation can be seen in Figure 2.

This time variation can be taken into account when designing a robust DFE. The channel identified during the training sequence is used for the nominal model $B_0(q^{-1})$. The stochastic uncertainty $\Delta B(q^{-1})$ will in this case consist of two parts. The first part reflects uncertainty in the identification of the channel model during the training sequence and the second part describes the effect of a time varying channel during the burst. In order to design the equalizer, the correlation matrix $P_{\Delta B}$

$$P_{\Delta B} = P_{TV} + P_{IU} \qquad (5)$$

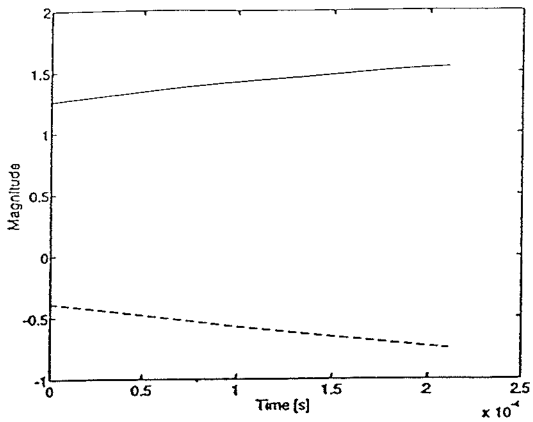

Figure 2: Example of the variation of a channel tap at carrier frequency 1800 MHz and mobile speed of 200 km/h. The time span is 58 symbols of 3.7μs each. Solid line: real part, dashed line: imaginary part.

with elements $E(\Delta b_i \Delta b_j^*)$ is required. Here the subscripts TV and IU stand for *time variation* and *identification uncertainty* respectively. The correlation matrix for the coefficients of the channel, P_B, with elements $E(b_i b_j^*)$, can be estimated on line by averaging over a number of bursts. The contribution from the time variation, P_{TV}, can then be related to P_B by

$$P_{TV} = 2P_B \left(1 - \frac{1}{T} \int_0^T J_0(2\pi f_c \frac{v_0}{c_0} \tau) d\tau \right) \quad . \qquad (6)$$

Here T is the time period averaged over, f_c is the carrier frequency, c_0 is the speed of light and v_0 is the speed of the mobile, which can be estimated. J_0 is a Bessel function of the first kind and order zero.

The covariance matrix P_{IU} can be calculated, by well known methods, from the training sequence and the estimated noise variance. The robust equalizer can now be designed, by using the resulting $P_{\Delta B}$ from (5), as described in [6],[7]. The performance gained with the robust DFE as compared to the nominal DFE can be seen in Figure 3 and Figure 4. The performance gain increases with increasing speed of the mobile, and is largest at large SNR's.

In the considered example, it turns out that the uncertainities in the coefficients of the channel are only weakly correlated. Therefore, one could design a good robust equalizer by only adding the sum of the diagonal elements of $P_{\Delta B}$ to the variance ratio $\rho \triangleq \sigma_d^2/\sigma_v^2$. This result is appealing because it is an action a designer would try first of all, in order to robustify the equalizer. Our method offers more than this. It constitutes a systematic approach which gives the right amount of robustification for each case, as well as a means to handle correlated channel parameters.

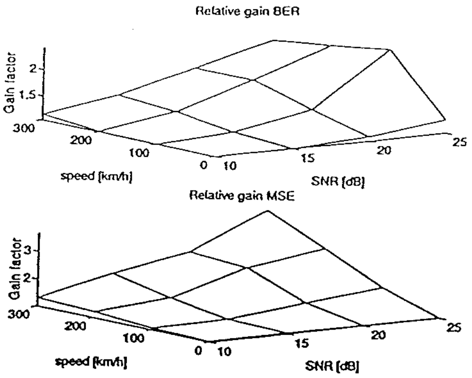

Figure 3: Performance gain, BER_{nom}/BER_{rob} and MSE_{nom}/MSE_{rob}, for the robust equalizer as compared to the nominal equalizer.

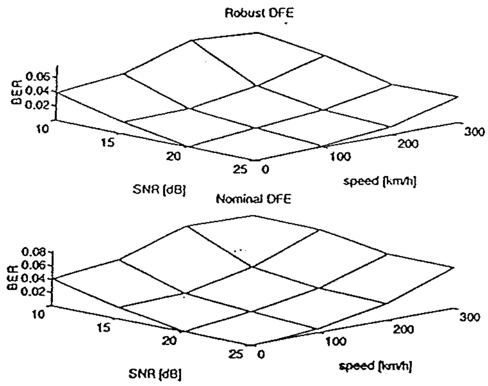

Figure 4: BER for the robust and the nominal DFE:s.

3. TRACKING OF FAST TIME VARYING PARAMETERS

If the carrier frequency is high and/or the symbol time is relatively long, time dispersion and fading may be of a size where robust equalizers can no longer cope. To preserve high-quality transmission in such channels, it is necessary to utilize adaptive channel equalization. Due to large and sudden phase jumps, which characterize mobile radio multipath channels, adaptive equalization should be based on *channel estimation.* (Equalizer parameters vary much faster than do the corresponding channel parameters.) The channel estimates are used in a Viterbi algorithm or in an equilizer. To use the Viterbi algorithm or the equalizers in an efficient way, predictions of future channel coefficients are needed. The length of the future window depends on the choice of decision delay in the Viterbi algorithm and on the choice of smoothing lag in the equalizer.

Let us consider a received sampled sequence $\{y_t\}_1^N$. It is generated by transmission of one data burst $\{d_t\}_1^N$ over a fading mobile radio channel, represented by an equivalent discrete time baseband (FIR) channel model (1), (2). Here, however, the parameters b_t are time varying. The channel model, of order nb, can be written as

$$\begin{aligned} y_t &= \varphi_t^* b_t + v_t \qquad\qquad t = 1, \ldots, N. \quad (7) \\ \varphi_t^* &= (d_t, d_{t-1}, \ldots, d_{t-nb}) \\ b_t &= (b_{0,t}, b_{1,t}, \ldots, b_{nb,t})^T \end{aligned}$$

where b_t denotes the channel coefficient vector and v_t is additive noise. In fading environments, the channel coefficients typically exhibit trend or quasi-periodic behaviour. The channel coefficients are often modelled as if they were subject to independent Rayleigh fading in urban areas. This type of prior knowledge is very valuable in the design of tracking algorithms.

A structured way of taking prior knowledge into account in the algorithm design, is to use a *hypermodel design approach.* In that approach, the channel coefficients are described by simple stochastic dynamic models. The models are choosen to capture typical parameter variations. Our hypermodel design results in tracking algorithms having the structure

$$\begin{aligned} \varepsilon_t &= y_t - \varphi_t^* \hat{b}_{t|t-1} \\ \hat{b}_{t+1|t} &= F(q^{-1})\hat{b}_{t|t-1} + G(q^{-1})\varphi_t \varepsilon_t, \end{aligned} \qquad (8)$$

where $F(q^{-1})$ and $G(q^{-1})$ are, in general, polynomial matrices. Above, $\hat{b}_{t|t-1}$ denotes the estimate of the channel coefficient vector at time t, given data up to time $t-1$. The LMS algorithm is obtained by setting F equal to the identity matrix and $G(q^{-1})$ to a scalar (2μ). As was mentioned above, efficient use of the Viterbi algorithm, or the equalizer, requires the use of future channel estimates. It is therefore essential to derive $k-$step predictions of the channel parameters

$$\hat{b}_{t+k-1|t-1} = P_k(q^{-1})\hat{b}_{t|t-1} \qquad k = 2, 3, \ldots, m_{delay},$$

where $P_k(q^{-1})$ are prediction filters and m_{delay} determines the length of the future estimation window required by eg the Viterbi algorithm.

We will now briefly address the problem of how to select hypermodels for different fading environments, and how to derive necessary filters for efficient tracking.

Coefficients of channels subject to Rayleigh fading, are well described by lightly damped second order autoregressive models

$$\bar{b}_{i,t} = 2r_d \cos\omega_d \bar{b}_{i,t-1} - r_d^2 \bar{b}_{i,t-2} + e_{i,t} \qquad (9)$$

where $e_{i,t}$ is white noise with zero mean. The pole locations are $r_d e^{\pm j\omega_d}$. The pole radius r_d reflects the damping and ω_d the dominating frequency of the parameter variations. The bar over b is used to emphasize that (9) is an approximation, and not a perfect stochastic dynamic model description of the true channel coefficient.

It is well known that the autocorrelation function of a channel coefficient subject to Rayleigh fading is such that $r_b(k) \sim J_o(2\pi f_D/f_s\, k)$, where $J_o(\cdot)$ is the Bessel function of the first kind and of zero order, f_D is the maximum Doppler frequency, f_s is the sampling rate and k is the time lag. The autocorrelation of (9) approximates the Bessel function very well over a time scale of a half-period π/ω_d. It is evident that the autocorrelation function depends strongly on the Doppler frequency. Since the Doppler frequency is related to the speed of the mobile, which might vary between consequtive bursts, hypermodels should be designed to cover different situations. ***Robust*** hypermodel design will therefore be of interest.

Assume that channel parameters vary according to (9). We then obtain

$$F(q^{-1}) = 2r_d \cos\omega_d - r_d^2 q^{-1}.$$

The filters G and P_k depend on the variances of v_t and e_t, as well as on the model structure itself. They will also depend on the distribution of the input data (the symbols). The main design parameters for tracking of channel parameters are F and the ratio between the variances of d_t and e_t. The current research is now focused on how to select polynomial matrices F and G in an optimal way. Here a Wiener filter approach [1], [2] is used. The results obtained so far are very promising.

4. OPTIMAL DIVERSITY COMBINING AND MULTIVARIABLE EQUALIZATION

Capacity limitations will always be an important issue in the design of cellular mobile telephone systems. One way of increasing the overall system capacity, is to make the cells smaller. However, this is expensive, and only possible to a certain extent. To combat the capacity problem, we propose the utilization of an extension of the well-known concept of antenna diversity, to increase the capacity of a single cell in the communication system. The main idea is shown (for the uplink case) in Figure 5. Two mobiles are allowed to use the same channel simultaneously. Considering a discrete-time, baseband model, the signal received at each of the two base station antennas is a linear combination of the messages transmitted from the two users. The corresponding discrete-time channels are indicated in the picture.

If the channel from one user to one base station antenna is *sufficiently different* from the channel from the same user to the *other* base station antenna, we could hope that it would be possible to receive different linear combinations of the two messages at the two base station antennas. If that is the case, it is possible to separate the two transmitted signals, by solving a system of linear equations.

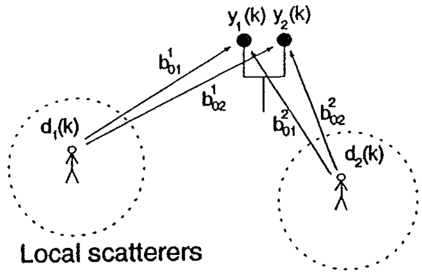

Figure 5: Scenario for the proposed multi-user diversity system (uplink). The letters labelling the paths between the transmitters and receivers represent filters, modelling the paths between them.

Unfortunately, there is no way to guarantee the two channels from a user to the base station to be sufficiently different. Instead, we have to rely on statistical properties of the channel. The degree of difference of the two channels can be described by the correlation of the two base station antennas. This is a well-known assumption when evaluating the performance of antenna diversity systems, and has also been the major parameter in our investigation of the method.

Although Figure 5 describes a scenario with two users and two antennas, our design equations are not limited to this situation. They can be used for arbitrary numbers of transmitters and receivers. For instance, with one transmitter and multiple receiver antennas, the proposed multivariable DFE can be seen as the MSE optimal diversity combiner when there is intersymbol interference. This combined diversity combiner-equalizer has been investigated in simulations, along with the multi-user scenario discussed above.

To formalize our ideas, we adopt a completely discrete-time model of the channel:

$$\mathbf{y}_t = \sum_{k=0}^{n_b} \mathbf{B}_k \mathbf{d}_{t-k} + \mathbf{v}_t$$

where $\mathbf{B}_k$ is a matrix of dimension $na|nd$, with na and nd being the number of antennas and mobiles respectively. The vectors $\mathbf{y}_t$, $\mathbf{d}_t$ and $\mathbf{v}_t$ represent the received signals, the transmitted messages and the noise, respectively. The equalizer parameters will be calculated based on a parametric model of the channel. We assume that such a model has been made available to us from eg identification of the channel.

To filter out the desired signals $\mathbf{d}_t$, we propose a simultaneous spatial combining and temporal equalization, using a *multivariable Decision Feedback Equalizer*

(DFE):

$$\hat{\mathbf{d}}_t = \sum_{k=0}^{n_S} \mathbf{S}_k \mathbf{y}_{t+\ell-k} - \sum_{k=0}^{n_Q} \mathbf{Q}_k \overline{\mathbf{d}}_{t-k-1}$$

where ℓ is the smoothing lag, and $\overline{\mathbf{d}}_t$ are previously detected symbols.

The matrix coefficients are calculated to minimize the MSE of the estimation error $\mathbf{z}_t$, ie

$$E[|\mathbf{z}_t|^2] = E[|\mathbf{d}_t - \hat{\mathbf{d}}_t|^2] \quad . \tag{10}$$

The filter coefficients minimizing (10) are involved in a set of coupled Diophantine equations. In essence, they can be obtained by solving a linear system of equations [5]. The discussion so far has concerned the uplink. Future systems with two antennas at the mobile are definitely feasible. Our algorithms are equally well suited for that case. This scenario has been investigated as well. For details see [5].

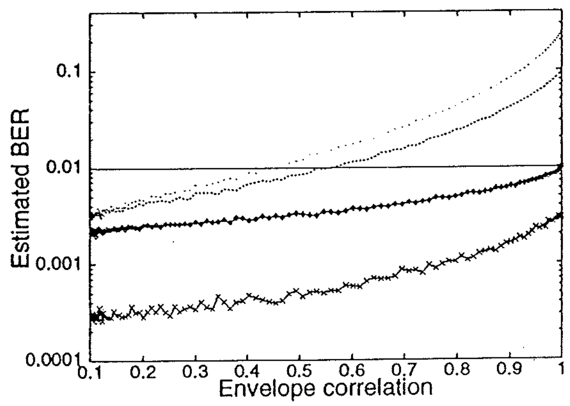

Figure 6: The BER for the six different situations listed below: single base station antenna (horizontal and solid), selection diversity (◇-solid), multivariable, single message (x-solid) and multivariable, two messages, uplink (dashed) and downlink (dotted).

To investigate the performance of the proposed multivariable approach, simulations have been carried out for the case of two receivers and two transmitters. The two antennas at the mobile have been assumed uncorrelated as they are supposed to be located in a Rayleigh fading environment and separated by, at least, half a wavelength. The correlation between the two base station antennas was varied, and the BER evaluated. A summary of the investigation is shown in Figure 6. The plot shows five curves. The solid line shows the BER of a system with one transmitter and one receiver. The two lower curves show a single transmitter scenario where there are two receiver antennas present and antenna diversity is used. The upper one of these two curves shows the BER when selection diversity is used whereas the lower curve shows the BER when the proposed multivariable DFE is used as the combined diversity combiner-equalizer previously mentioned. Note that this BER is *very* low. Finally, the two upper curves show the BER when the multivariable DFE is used with two transmitters and two receivers: the uppermost one showing the downlink and the lower the uplink. Note that twice the number of bits has been transmitted in these two cases. In all the above cases, the channel had two taps. The SNR was defined as

$$\mathrm{SNR} = \frac{E[|b_{0i}^k|^2 + |b_{1i}^k|^2]}{E[|v_i|^2]} \qquad i,k = 1,2$$

The integer i labels the receiver and k the transmitter. In the simulation depicted in Figure 6, binary signalling with SNR 10 dB was used. For each envelope correlation, 1000 channels were simulated, each with 400 symbol pairs. The channels were subject to independent Rayleigh fading. A smoothing lag $\ell = 1$ was used in the equalizer.

The ideas and algorithms described above are discussed thoroughly in [5].

References

[1] A. Ahlén and M. Sternad, "Wiener filter design using polynomial equations". *IEEE Trans. Signal Processing*, vol 39, pp 2387–2399, 1991.

[2] A. Ahlén and M. Sternad, "Derivation and Design of Wiener Filters Using Polynomial Equations." In Ed's C.T. Leondes: *Academic Press Theme Volumes on Digital Signal Processing Techniques and Applications*, Academic Press, 1994.

[3] M. Sternad and A. Ahlén, "The structure and design of realizable decision feedback equalizers for IIR channels with colored noise". *IEEE Trans. Information Theory*, vol 36, pp 848–858, 1990.

[4] M. Sternad and A. Ahlén, "Robust filtering and feedforward control based on probabilistic descriptions of model errors". *Automatica*, vol 29, pp 661-679, 1993.

[5] C. Tidestav, "Optimum diversity combining in multi-user digital mobile telephone systems" *Master Thesis*, UPTEC 93094E, Department of Technology, Uppsala University, 1993.

[6] M. Sternad, A. Ahlén and E. Lindskog, "Robust decision feedback equalizers". ICASSP 1993, Minneapolis, MN, USA, April 27-30 1993, vol III, pp 555-558.

[7] E. Lindskog, M. Sternad and A. Ahlén "Designing decision feedback equalizers to be robust with respect to channel time variations". Nordic Radio Society Symposium on disturbance resistant radio and radar, Uppsala, November 10–11,1993.