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ROBUST FEEDFORWARD CONTROL BASED ON PROBABILISTIC DESCRIPTIONS OF MODEL ERRORS

Mikael Sternad and Anders Ahlén

Systems and Control Group, Uppsala University
P.O. Box 27, S–751 03 UPPSALA, Sweden

We will present a simple and flexible method for building performance robustness into the design of linear filters for open loop problems. Such problems include signal estimation, state estimation and feedforward control. The key is a novel design method for Wiener filters. It can be regarded as a generalization of Kučera's polynomial approach. A detailed presentation of this method is given in Sternad and Ahlén (1992,1993). Here, we briefly describe the design of robust feedforward links from disturbances or from command signals.

Can the performance robustness of a feedforward controller be improved? For any such compensator, Bodes relative sensitivity function is = 1. This means that a $x\%$ parameter deviation will result in a $y\%$ deviation of the transfer function at frequency ω_i, regardless of the applied filter. However, the *effect* of the $y\%$ deviation on e.g. the step response will very much depend on the magnitude of the transfer function at ω_i. (The *absolute* sensitivity function describes this effect, while the commonly used *relative* one misses it completely!) When the performance of a nominal LQG controller is sensitive to model errors, our robustified control achieves a radical (absolute) sensitivity reduction. There is only a small performance deterioration in the nominal case, if the assumed modelling errors are not extremely large.

Modelling errors will be described by sets of systems, parametrized by random variables with known covariances. Robust control is then obtained by minimizing a quadratic criterion, averaged both with respect to model errors and the noise. A polynomial solution, based on averaged spectral factorizations and a Diophantine equation, is presented.

Robust design turns out to be no more complicated than the design of an ordinary LQG feedforward controller. The proposed method is very simple to use, and it also has two other advantages. First, probabilistic descriptions of model uncertainties may have *soft bounds*. These are more readily obtainable in a noisy environment than the hard bounds required for e.g. minimax design. Furthermore, not only the range of uncertainties, but also their likelihood is taken into account; common model deviations will have a greater impact on a controller design than do very rare "worst cases". The conservativeness is thus reduced, compared to worst case design.

Let us briefly describe the model structure and design method. Consider a stable discrete–time model, describing a scalar output to be controlled $y(t)$, input $u(t)$ and a measurable signal $w(t)$:

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t-k) + \frac{D(q^{-1})}{F(q^{-1})}w(t-d) \quad ; \quad w(t) = \frac{G(q^{-1})}{H(q^{-1})}v(t) \; .$$

Here, $v(t)$ is white, with unit variance. The possibly uncertain system B/A may be open loop, or could represent a plant controlled by a given feedback regulator. To take uncertainty into account, consider a model of the whole class of possible systems. We use the polynomial parametrization

$$\frac{B}{A} = \frac{B_0}{A_0} + \frac{B_1 \Delta B}{A_1} \quad ; \quad \frac{D}{F} = \frac{D_0}{F_0} + \frac{D_1 \Delta D}{F_1} \quad ; \quad \frac{G}{H} = \frac{G_0}{H_0} + \frac{G_1 \Delta G}{H_1} \; .$$

Here, B_0/A_0 etc. represent stable and known *nominal* models, while $B_1 \Delta B/A_1$ etc. are members of model error classes, with $B_1(q^{-1})$ and $A_1(q^{-1})$ being fixed. The coefficients of ΔB, ΔD and ΔG are zero mean stochastic variables. They have known positive semidefinite covariance matrices $\mathbf{P}_{\Delta B}$ etc. All denominators are stable. This parametrization is flexible, without sacrificing linearity in stochastic parameters. We present examples of how such *stochastic additive error models* can be tailored to fit the size and the spectral properties of the model uncertainty.

A stable and causal feedforward filter, operating on $w(t)$, is to be designed, in order to minimize an infinite horizon quadratic criterion, with input penalty $\rho \geq 0$

$$J = \bar{E}E(y(t)^2 + \rho u(t)^2) \; .$$

Above, $\bar{E}$ and E denote expectation with respect to model error distributions and noise, respectively.

In a *disturbance measurement feedforward* problem, $w(t)$ represents the disturbance. In a *command feedforward* problem, $w(t)$ is a command signal and $(G/H)v(t)$ is a stochastic model, describing its second order properties. A servo filter is then to be designed, so that $(B/A)u(t-k)$ optimally follows a response model $-(D/F)w(t-d)$. The solution to both these problems is given by two "averaged" spectral factorizations and a Diophantine equation. It is presented below.

For any polynomial $P(q^{-1})$, let $P_* = P(q)$. Calculate a stable and monic polynomial spectral factor $\tilde{G}(q^{-1})$, and a scalar s, from

$$s\tilde{G}\tilde{G}_* = \bar{E}(GG_*) = G_0 G_{0*} H_1 H_{1*} + \bar{E}(\Delta G \Delta G_*) G_1 G_{1*} H_0 H_{0*} \; .$$

(With no uncertainty, $\tilde{G} = G_0$.) Then, calculate another stable and monic spectral factor $\beta(q^{-1})$, and a scalar r, from

$$r\beta\beta_* = B_0 B_{0*} A_1 A_{1*} + \bar{E}(\Delta B \Delta B_*) B_1 B_{1*} A_0 A_{0*} + \rho A_0 A_{0*} A_1 A_{1*} \; .$$

Now, a stable and causal feedforward filter, minimizing J, is given by

$$u(t) = -\frac{Q_1 A_0 A_1}{\beta F_0 \tilde{G}} w(t)$$

where $Q_1(q^{-1})$, together with $L_*(q)$, is the unique solution to the Diophantine equation

$$q^{-d+k} B_{0*} A_{1*} D_0 \tilde{G} = r\beta_* Q_1 + qF_0 H_0 H_1 L_* \quad .$$

We call the filter above a *cautious Wiener filter*. Averaged factors $\bar{E}(\Delta B \Delta B_*)$ and $\bar{E}(\Delta G \Delta G_*)$ are symmetric polynomials. Their coefficients are readily obtained from $\mathbf{P}_{\Delta B}$ and $\mathbf{P}_{\Delta G}$, by summation diagonalwise. Note that only second order moments or, equivalently, the frequency domain variances of the transfer functions ($\bar{E}(\Delta B \Delta B_*) B_1 B_{1*} / A_1 A_{1*}$ etc.) need to be known. For identified models, the variance in the frequency domain, corresponding to a given covariance matrix for estimated polynomial coefficients, can be calculated approximately. (This is done in e.g. the System Identification Toolbox for Matlab.) Using the stochastic embedding approach, developed by G. C. Goodwin, the freqency domain variance of modelling errors can be estimated directly from the data, also when the model structure is incorrect. See Goodwin et al. (1992). If no data is available, error models can still be used as robustness "tuning knobs" for the regulator design.

Design equations for the nominal case (without uncertainty) can be derived for problems with frequency–dependent weights in the criterion and noise on $w(t)$. By comparing the solution above to such design equations, some important conclusions can be drawn.

- The uncertainty $G_1 \Delta G / H_1$ of the disturbance or reference generator has a similar impact on the design as does a *coloured measurement noise* on $w(t)$.
- The uncertainty $B_1 \Delta B / A_1$ of the system has a similar impact as would a *frequency weighted input penalty* in the criterion. Also, note that if $|1/A_1(e^{i\omega})|$ has a resonance peak at frequecy ω_1, indicating large uncertainty, the feedforward controller will have low gain at that frequency.
- Somewhat surprisingly, uncertainty in D/F has no effect at all on the design.

The intuitive notion that assumption of measurement noise on $w(t)$ and increase of the input penalty ρ would improve the performance robustness of a nominal design, is thus supported. With the above equations, the robustification can be tailored more exactly to the expected type and amount of uncertainty.

References:

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