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THE USE OF DISTURBANCE MEASUREMENT FEEDFORWARD IN LQG SELF-TUNERS

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Abstract: An explicite adaptive regulator with disturbance measurement feedforward is presented. It is based on a polynomial LQG design. In feedback regulators, the optimization of a feedforward filter involves the solution of only one additional linear polynomial equation. The regulator is designed to handle shape-deterministic disturbances, such as steps, ramps and sinosoids, as well as stochastic disturbances. Computational aspects, the computational complexity and the robustness against unmodelled dynamics are discussed. It is argued that the use of feedforward can improve not only the disturbance rejection, but also the stability robustness of an LQG feedback regulator.

Keywords: Feedforward control; disturbance rejection; linear optimal regulator; LQG; adaptive control; robustness.

1 Introduction

A feedforward regulators utilizes measurements of important disturbances. When there is a delay between disturbance and controlled output, the regulator reacts on the disturbance before it begins to affect the controlled variable. Complete disturbance cancellation may sometimes be achieved. Addition of feedforward filters to feedback regulators is a simple way to improve control performance, at moderate extra computational cost.

LQG optimization is a good framework for the design of combined feedback and feedforward regulators. It provides tradeoffs between input energy and disturbance rejection. Control of systems with input delays or non-mimimum phase dynamics becomes straightforward. Several alternative approaches do however exist, such as generalized minimum variance control (GMV). See Åström and Wittenmark (1973), Clarke and Gawthrop (1979), Allidina et. al. (1981) and Tahmassebi et. al. (1985). Compared to adaptive algorithms based on infinite horizon LQG criteria, GMV often attains inferior asymptotic performance. This is the case in particular for non-minimum phase systems, cf. Modén and Söderström (1982) and Sternad (1987). The quest for improved performance has led to the modification of GMV into Generalized Predictive Control (GPC), Clarke et. al. (1987), Perez and Kershenbaum (1986). As the prediction horizon increases, the performance of GPC approaches that of infinite horizon LQG control from below.

LQG optimization can be based on polynomial equations (Kučera 1979). The polynomial equations approach to the design of combined feedback-feedforward regulators has received considerable interst recently. See Peterka (1984), Šebek et. al. (1988) and Sternad and Söderström (1988). LQG self-tuners with disturbance measurement feedforward have been proposed by Sternad (1986),(1987) and Hunt et. al. (1987).

The purpose of this paper is to discuss the following aspects related to adaptive feedback/feedforward control based on LQG design:

- While LQG design is based on a stochastic disturbance description, random step sequences, ramp sequences and sinusoids can also be handeled. Disturbance measurement feedforward can be combined with integrating feedback, although some care must be taken with the regulator calculation and implementation.

- Feedforward control may be used to improve the stability robustness of feedback regulators. Assume that a given amount of disturbance rejection is desired. The regulator design is based on an uncertain and/or underparametrized model. When most of a disturbance can be eliminated by feedforward, the high-frequency gain of the feedback can be reduced. The feedback can be designed to maintain robust stability, rather than high disturbance rejection. (With an incorrect model, the feedforward control performance will of course be non-ideal, but this can never make the system unstable.)

- LQG feedback control laws place some poles (the 'observer poles') at disturbance model zeros. It will be argued that this is a bad idea when used in adaptive control. With other (suboptimal) observer poles, the robustness can be increased, often with only a small deterioration of the disturbance rejection.

The paper is organized as follows. For the control problem defined in Section 2, the polynomial LQG solution is presented in Section 3. A self-tuning implementation is described in Section 4. Some user-choices which affect the robustness of the control law are discussed in Section 5.

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2 The control problem

Let the plant be described by the following linear discrete time model

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + D(q^{-1})w(t-d) + C(q^{-1})e(t) \quad (1)$$

where the output $y(t)$, input $u(t)$, measurable disturbance $w(t)$ and unmeasurable disturbance $e(t)$ are all scalar signals. No unstable common factors are present in $A(q^{-1})$ and $B(q^{-1})$. All polynomials in the backward shift operator q^{-1}, except $B(q^{-1})$, are monic.

The disturbances $w(t)$ and $e(t)$ are modelled by

$$\begin{aligned} w(t) &= \frac{G(q^{-1})}{H(q^{-1})}v(t) \\ e(t) &= \frac{1}{F(q^{-1})}n(t) \end{aligned} \quad (2)$$

where $H(q^{-1}) = H_S(q^{-1})H_U(q^{-1})$. We assume $v(t)$ and $n(t)$ to be mutually uncorrelated and zero mean. They are white noises or random spike sequences. While $C(q^{-1})$, $G(q^{-1})$ and $H_S(q^{-1})$ are assumed to be stable, $H_U(q^{-1})$ and $F(q^{-1})$ have all their zeros on the unit circle. The disturbance models thus include

1. *Stationary stochastic disturbances.* (F or $H_U = 1$)
2. *Drifting stochastic disturbances.* If $w(t)$ e. g. has stationary increments, it is modelled by $H_U = 1 - q^{-1}$ and a white noise v(t).
3. *Shape-deterministic* or piecewise deterministic signals, such as random step sequences, ramp sequences or sinusoids which occationally change magnitude or phase. A stationary random spike sequence, such as a Bernoulli-Gaussian sequence is then a reasonable model for $v(t)$ or $n(t)$.

The goal is to minimize an infinite horizon criterion

$$J = \lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N} Ey(t)^2 + \rho E(\tilde{\Delta}(q^{-1})F(q^{-1})u(t))^2 \quad (3)$$

The input penalty $\rho \geq 0$ and the polynomial $\tilde{\Delta}(q^{-1})$ are chosen by the designer. They define a frequency dependent tradeoff between input energy and disturbance rejection. Note that the choice of input filter is not completely free: the factor $F(q^{-1})$ must be present whenever $e(t)$ is described by an unstable model. If $e(t)$ e. g. is a drifting stochastic signal, a drifting input $u(t)$ will be needed. To keep the criterion finite, the input must then be filtered by $F(q^{-1}) = 1-q^{-1}$ in (3). For the same reason, $H_U(q^{-1})$ must be a factor of $\tilde{\Delta}(q^{-1})F(q^{-1})$, unless it is a factor of $D(q^{-1})$.

It can be shown (Sternad 1987) that the optimal linear regulator structure with feedback and feedforward is given by

$$R(q^{-1})F(q^{-1})u(t) = -\frac{Q(q^{-1})}{P(q^{-1})}w(t) - S(q^{-1})y(t) \quad (4)$$

See Figure 1. The polynomial $P(q^{-1})$ is required to be stable. Note that the factor $R(q^{-1})F(q^{-1})$ is present in both the feedback and feedforward signal paths. The filtering by $F(q^{-1})$ is consistent with the internal modelling principle (Francis and Wonham 1976). When $F(q^{-1}) = 1 - q^{-1}$, we have an integrating regulator with feedforward term.

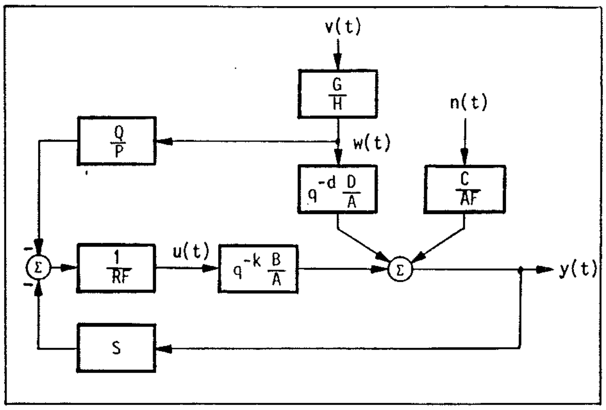

Figure 1. The system and regulator structure.

In Sternad and Söderström (1988), a polynomial equation was presented by which the feedforward filter $\{P, Q\}$ can be optimized, given *any* stabilizing feedback $\{R, S\}$. The use of PID-control, pole placement feedback or no feedback at all are some examples. Based on that equation, an LQG self-tuning feedforward filter has been designed which complements existing (adaptive or non-adaptive) feedback regulators (Sternad 1987).

In this paper, we will however discuss the optimization of the total regulator (4), with respect to (3). The design consists of a simple two-step procedure: the feedback $\{R, S\}$ is first optimized with respect to the unmeasurable disturbance $e(t)$, using well known methods (Kučera 1979, Peterka 1984). The feedforward filter $\{P, Q\}$ is then calculated so that $w(t)$ is rejected in an optimal way. This separability is made possible by the use of the regulator structure (4), and by the (assumed) noncorrelation between $w(t)$ and $e(t)$.

3 The optimal regulator

In order to obtain a convenient notation, substitute z for q^{-1} and define , for any polynomial $P(z)$, $P_* = P(z^{-1})$ and $\overline{P} = z^{np}P_*$. The polynomial arguments will in general be omitted in the following.

Introduce the spectral factorization

$$r\beta\beta_* = BB_* + \rho AF\tilde{\Delta}\tilde{\Delta}_*F_*A_* \quad (5)$$

where r is a positive scalar, and β is a stable monic polynomial in z with degree $n\beta$. To assure that a stable β exists, we require B to have no zeros on the unit circle when $\rho = 0$. When $\rho > 0$, B and $AF\tilde{\Delta}$ should have no common factors with zeros on the unit circle.

Theorem 1

Assume that H_U is a factor of $\tilde{\Delta}FD$, and that a stable spectral factor β in (5) exists. The regulator (4) then attains the global minimum value of (3), under the constraint of stability, if $\{P, Q, R, S\}$ are calculated as follows:

Let $R_*(z^{-1}), S_*(z^{-1})$ and $X(z)$ be the minimum degree solution with respect to X of the coupled polynomial equations

$$r\beta R_* - z^{-k+1}B_*X = \rho\tilde{\Delta}\tilde{\Delta}_*AFC_* \quad (6)$$
$$r\beta S_* + zA_*F_*X = z^kBC_* \quad (7)$$

where β is the stable spectral factor from (5).

Let $P = G$, and let $Q_*(z^{-1})$ and $L(z)$ be the solution of

$$z^{-d+1}D_*F_*G_*X = r\beta Q_* + C_*H_*zL \quad (8)$$

□

Outline of proof: Refer to Sternad and Söderström (1988). It is straightforward to show that all results there apply also when $H_U \neq 1$, if H_U is a factor of $\tilde{\Delta}FD$. Multiply (1) by F: $(AF)y(t) = q^{-k}B(Fu(t)) + q^{-d}(DF)w(t) + Cn(t)$. Since $n(t)$ is a white stationary random sequence, Theorem 4 in Sternad and Söderström (1988) can be applied to this model structure. Thus, the signal $Fu(t)$ is used instead of $u(t)$ in regulator and criterion. AF is substituted for A and DF for D in all equations. The Theorem then follows.

Remarks:
Properties of the feedback:

The variables in (6) and (7) have degrees ($nx \triangleq \deg X$ etc.)

$$\begin{aligned} nx &= n\beta + k - 1 \\ ns &= \max\{na + nf - 1, nc - k\} \\ nr &= \begin{cases} \max\{nb + k - 1, nc + \deg\tilde{\Delta}\} & \text{if } \rho \neq 0 \\ nb + k - 1 & \text{if } \rho = 0 \end{cases} \end{aligned} \quad (9)$$

Multiply (6) by A_*F_* and (7) by $z^{-k}B_*$ and add them. Optimal feedback is then seen to imply pole placement in βC:

$$\beta C = AFR + z^kBS \quad (10)$$

If A and B had no common factors, an optimal feedback could be calculated from the implied pole placement equation (10). With common factors, this will not be possible (Kučera 1984). The equations (6) and (7) however give the correct solution, as long as the common factors are stable.

Assume $F(q^{-1}) = 1 - q^{-1} \triangleq \Delta(q^{-1})$. According to (2), $F = \Delta$ models the dynamics of step and Wiener process disturbances $e(t)$. In the regulator, $F = \Delta$ represents integration. Nothing prevents us from using integration i. e. to set $F = \Delta$ in (4)-(8) also when $e(t)$ is stationary. We cannot then attain the minimal criterion value, because the regulator has incorrect structure, but it may be advantageous to use integration anyway. When the feedforward filter is imperfectly designed, static control errors will then be taken care of by the feedback.

The feedforward controller calculation:

Note that the solution of only one additional Diophantine equation, namely (8), is needed for optimizing a feedforward filter. Since β (stable) and $z^{nc}C_*z^{nh}H_* = \overline{CH}$ (unstable) cannot have common factors, (8) is always solvable. The degrees of $Q_*(z^{-1})$ and $L(z)$ are uniquely defined by the requirement that they should cover the maximal occuring powers in z^{-1} and z, respectively, in (8):

$$\begin{aligned} nQ &= \max\{nd + nf + ng + d, nc + nh\} - 1 \\ nL &= \max\{0, k - d\} + n\beta - 1 \end{aligned} \quad (11)$$

The polynomial L is not used in the controller. The delay d affects the achievable control quality significantly. It can be shown that application of feedforward can always improve the control performance when $d > 0$, compared to feedback from $y(t)$ only. The improvement is a nondecreasing function of d. It is advantageous to place the auxiliary $w(t)$-sensor so that the disturbance is captured as early as possible, i. e. d is large.

The solution can be generalized to multiple measurable disturbances. If the measurement $w(t)$ is influenced by the input $u(t-n), n > 0$, this effect could be subtracted internally, inside the regulator. See Sternad (1986),(1987).

Numerical aspects:

A common special case is when the measurable disturbance is drifting or of random step type, and an integrating regulator is used. Then, $H_U = F = \Delta$. Since Δ_* becomes a factor of both the left and right term in (8), it must also be a factor of Q_*. With $Q = Q_1\Delta$, equation (8) is reduced to

$$z^{-d+1}D_*G_*X = r\beta Q_{1*} + C_*H_{S*}zL \quad (12)$$

In this case, the controller (4) must be modified slightly. It can be implemented in differential form, using an explicite differentiation of the measurable disturbance $w(t)$:

$$\begin{aligned} R(\Delta u(t)) &= -\frac{Q_1}{G}(\Delta w(t)) - Sy(t) \\ u(t) &= u(t-1) + \Delta u(t) \end{aligned} \quad (13)$$

Alternatively, one can use a structure with the feedforward filter separated from the differentiation:

$$Ru(t) = -\frac{Q_1}{G}w(t) - \frac{S}{\Delta}y(t) \quad (14)$$

If (8) were used, small numerical errors and finite wordlenght effects would cause $Q \neq Q_1\Delta$. This could lead to large errors in the low-frequency gain of the feedforward filter $-Q/R\Delta P$ in (4). Design from (12) and realization according to (13) or (14) avoids such problems. Equation (8) must however be used in the general case, when $H_U \neq F$.

The regulator (4) or (13)/(14) must be realized minimally, as a single dynamical system having two inputs and one output. A reliable algorithm for spectral factorization can be found in Kučera (1979). It is iterative, requiring typically

3-10 iterations. The coupled equations (6),(7) represent an over-determined set of simultaneous equations in the coefficients of R, S and X. The system will however have a unique solution with the polynomial degrees (9). (Some equations are linear combinations of the others.) This (exact) solution can be found by computing the least-squares solution to the overdetermined system.

4 The LQG self-tuner

For systems with unknown or time-varying dynamics, an explicite LQG self-tuner has been developed (Sternad 1987). It is based on recursive system identification using the Recursive Prediction Error Method (RPEM), Ljung and Söderström (1983). The controller is redesigned periodically according to Theorem 1. A similar algorithm has been suggested by Hunt et. al. (1987). Upper bounds on all polynomial degrees are assumed known, together with the unstable disturbance model factor $F(q^{-1})$. The regulator, complemented with a servo filter, is summarized in Table 1 below.

1. Read new samples of $y(t), w(t)$ and a set-point $r(t)$.
2. Update models of $y(t)$ and $w(t)$ with the structure
$$\hat{A}y(t) = \hat{B}u(t) + \hat{D}w(t) + \hat{C}\varepsilon_y(t) \qquad (15)$$
$$\hat{H}w(t) = \hat{G}\varepsilon_w(t) \qquad (16)$$
using two RPEM routines for single output systems.
3. Compute r and $\beta(q^{-1})$ from the spectral factorization.
4. Determine $R(q^{-1}), S(q^{-1})$ and $X(q^{-1})$ from (6),(7).
5. Set $P(q^{-1}) = \hat{G}(q^{-1})$, and calculate $Q(q^{-1})$ and $L(q^{-1})$ from (8).
6. If needed, design a servo filter $T(q^{-1})/E(q^{-1})$.
7. Compute the control action:
$$RFu(t) = -\frac{Q}{\hat{G}}w(t) - Sy(t) + \frac{T}{E}r(t) \qquad (17)$$
8. Shift all data vectors, and go to step 1.

Table 1. An LQG feedback-feedforward self-tuner.

Remarks:

Step 2. The regressors of the model (15) are filtered by
$$\frac{F(q^{-1})}{N(q^{-1})} \qquad (18)$$
where $N(q^{-1})$ is a stable polynomial. Filtering by $F(q^{-1})$ is necessary to avoid biased estimates. With $N(q^{-1})$, the filter can be modified to improve the estimation accuracy in important freqency regions. The estimates of G and C must be projected into stable regions. The usual precautions of a control error dead-zone and covariance monitoring have been implemented. They guard against estimator wind-up and identification based on insufficient information.

Step 6. The servo filter T/E has been designed by cancelling poles and stable zeros so that a reference model $y_m(t) = (q^{-k}B_m/A_m)r(t)$ is approximated. This works well, but results in a rather high-order filter. Other approaches are discussed in Sternad (1987).

Step 7. When appropriate, the regulator (13) or (14), based on equation (12), should be used instead of (17).

2:	Identification		$37n^2$	$+36n$
3:	Spect. fact. (per iteration)		$3n^2$	$+3n$
4:	Feedback optimization	$36n^3$	$+87n^2$	$+135n$
5:	Feedforward opt.	$9n^3$	$+14n^2$	
7:	Control			8n

Table 2. The approximate number of mult-add operations required per sample, assuming all model polynomials to have equal degree n. A least squares solution is computed in step 4.

The computational burden of this algorithm is one order of magnitude higher than for GMV. See Table 2. With modern microcomputers and signal processors, this should be no significant restriction in most control applications. There is no need to recalculate the regulator at each sample. Steps 3-6 can be placed in a background process which provides a new regulator every m'th sample. For $m = 5 - 10$, this results in only a small degradation of the adaptation transient when the system dynamics changes. (It has recently been shown by Shimkin and Feuer (1988) that it can in fact be advantageous to update the regulator infrequently.)

Example 1

Let $(1/1 - q^{-1})v(t)$ be a square wave disturbance with unit amplitude and period 60. It disturbs the system
$$(1 - 0.5q^{-1})y(t) = (b_2 + b_3q^{-1})u(t-2) + (1 + 2q^{-1})w(t-1)$$
$$w(t) = \frac{1 - 0.3q^{-1}}{1 - 0.9q^{-1}}\left(\frac{1}{1 - q^{-1}}v(t)\right)$$
Comparing with (2), we have $G = 1 - 0.3q^{-1}$, $H_S = 1 - 0.9q^{-1}$ and $H_U = 1 - q^{-1}$, while $e(t) = 0$. The polynomial $b_2 + b_3q^{-1}$ changes from $1 + 0.1q^{-1}$ to $0.5 + 0.05q^{-1}$ at time 300. A correctly parametrized LQG self-tuner is applied, using an input penalty $\rho = 0.5$ and $\tilde{\Delta} = 1 - q^{-1}$, with $F = 1$, i. e. no integration. A forgetting factor of 0.98 is used. After an initial open loop identification period of 20 samples, the regulator quickly converges.

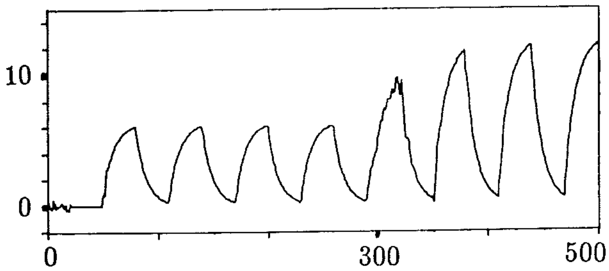

Figure 2. The input $u(t)$ in Example 1.

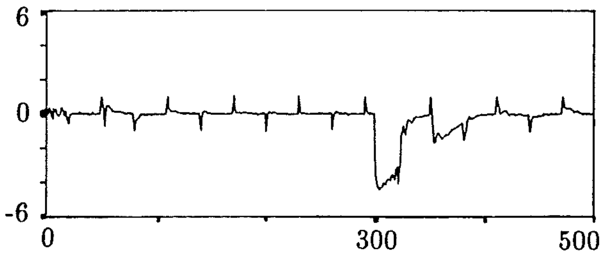

Figure 3. The controlled output $y(t)$ in Example 1. The disturbance $w(t)$ is cancelled almost completely, although the delay difference $k - d = 1$ prevents perfect cancellation. At $t = 300$, the system gain is halved. At $t = 400$, the control performance has recovered.

□

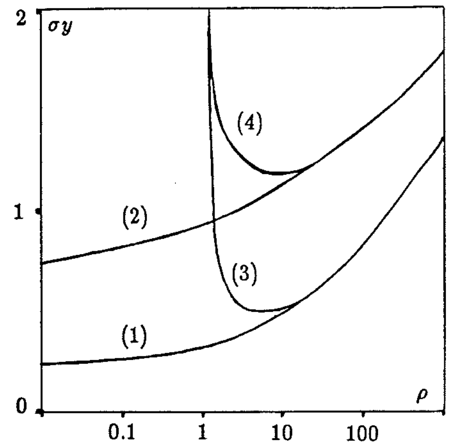

Figure 4. The output standard deviation σy in Example 2, as a function of the input penalty ρ.
(1): Feedback and feedforward. $\hat{B}$ of correct order 1.
(2): Feedback only. $\hat{B}$ of correct order 1.
(3): Feedback and feedforward. $\hat{B}$ of order 0.
(4): Feedback only. $\hat{B}$ of order 0.

5 Robustness-improving user choices

The robustness against unmodelled dynamics of a self-tuner is affected by properties of both the estimator and the control law. Simple considerations regarding the LQG control strategy, which in general improve the robustness of a self-tuner, are illustrated by the following example.

Example 2.

The system

$$(1 - 1.2q^{-1} + 0.52q^{-2})y(t) = q^{-2}(1 + 0.8q^{-1})u(t)$$
$$+ q^{-2}(1 - 0.2q^{-1})w(t) + (1 - 0.2q^{-1})e(t)$$

is affected by measurable and unmeasurable drifting stochastic disturbances $(1 - q^{-1})w(t) = v(t), (1 - q^{-1})e(t) = n(t)$. The white noises $v(t)$ and $n(t)$ have standard deviations 0.3 and 0.1, respectively. Thus, the largest disturbance is measurable, and $H_U = N = \Delta$.

The control error standard deviation was measured (after convergence) in simulation runs with four self-tuners. Integrating regulators with the structure (13), and with $r(t) = 0$ and $\tilde{\Delta} = 1$ were used. The results are shown in Figure 4, as functions of the input penalty ρ. Curve (1) represents the performance of LQG feedback and feedforward. When $\rho \to 0$, the disturbance $w(t)$ is cancelled completely by the feedforward control action. When only feedback is used, curve (2) is obtained. The disturbances $w(t)$ and $e(t)$ are then treated as one unmeasurable noise. The performance is obviously degraded without disturbance measurement. Correctly parametrized models were used in these experiments. Curve (3) and (4) result if an underparametrized $\hat{B}$ is used ($nb = 0$ instead of $nb = 1$). For input penalties $\rho \leq 1$, the closed-loop system then becomes unstable.

The reason for this behaviour is explained by Figures 5 and 6. Figure 5 shows Bode magnitude plots of some under-parametrized models obtained at the end of the simulation runs. Compare them with the true system. The gain at high frequencies is under-estimated, because the system zero cannot be modelled. For low ρ, the regulators have large feedback gains at high frequencies, cf. Figure 6. (This is often the case for minimum variance regulators.) The combination of large feedback gain and an incorrect model at high frequencies leads to instability.

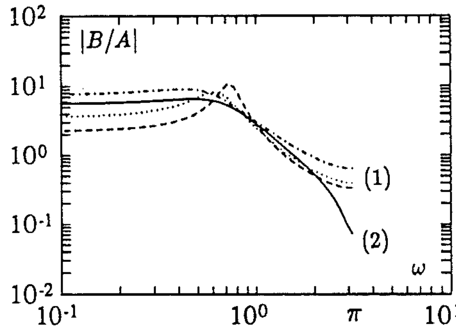

Figure 5. (1): Transfer function magnitudes for some under-parametrized models. (2): The true system.

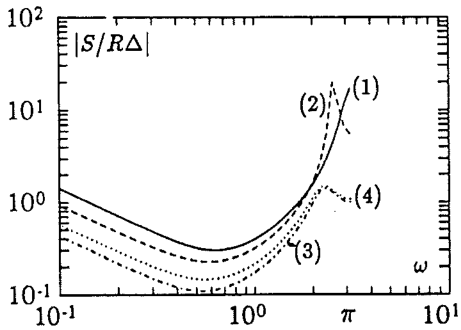

Figure 6. Transfer function magnitudes of feedback filters.
(1): $\rho = 0$. (2): $\rho = 0.5$. (3): $\rho = 10$.
(4): $\rho = 0.5$, with pole placement in C_O.

One way of reducing the high-frequency feedback gain is to modify the polynomial $C(q^{-1})$ used in (6)-(7). Instead of the estimate $\hat{C}$, a fixed polynomial $C_O = (1 - 0.5q^{-1})^2$ was used. This decreased the feedback high-frequency gain (cf. (4) in Figure 6). The performance for high ρ deteriorated somewhat, but the robustness for low ρ improved (Figure 7). The regulator now became stable for $\rho \geq 0.2$.

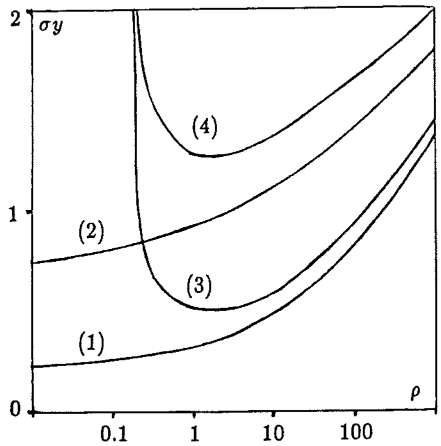

Figure 7. The output standard deviation σy versus the input penalty ρ, when estimated polynomials $\hat{C}$ and the fixed prespecified $C_O = (1-0.5q^{-1})^2$ are used for pole placement.
(1): Feedback and feedforward. $\hat{C}$ used. $\hat{B}$ of order 1.
(2): Feedback only. $\hat{C}$ used. $\hat{B}$ of order 1.
(3): Feedback and feedforward. C_O used. $\hat{B}$ of order 0.
(4): Feedback only. C_O used. $\hat{B}$ of order 0.

□

Let us summarize three robustness-enhancing user-choices:

- By *increasing ρ from zero*, the control signal variations and the high-frequency gains of both feedback and feedforward filters are reduced. Large reductions can often be achieved with only minor deterioration of the disturbance rejection. This increases the robustness against unmodelled high-frequency dynamics. Problems with hidden intersample output oscillations are also avoided.

- *Use of feedforward* can increase the stability robustness. This is possible when high disturbance rejection is required, and the main system disturbance is measurable. See e. g. Figure 4: if σy below 1 is required, this could be attained in the ideal case by a high gain (low ρ) feedback. Instability would however result in the underparametrized case. With both feedback and feedforward, a low gain regulator ($\rho = 10 - 300$) can be used. It easily attains the required performance, also in the underparametrized case.

- With LQG control, poles are placed in βC, cf. (10). The polynomial C could be interpreted as the observer dynamics in a state space formulation. Use of *a fixed prespecified observer polynomial* C_O, with $1/C_O$ being low-pass, has several advantages. While the zeros of β can be modified via ρ, we do not have any control over the zero locations of C in the true system. $1/C$ might very well be an extreme high-pass filter. Furthermore, the coefficients of C are the hardest ones to estimate. Estimated C-polynomials sometimes tend to contain a factor $1-q^{-1}$, which gives bad pole placement. (This happens when regressors are differentiated to avoid bias due to a non-zero mean disturbance $e(t)$, but $e(t)$ is stationary.) With a suboptimal pole placement βC_O, the feedback disturbance rejection may deteriorate. This matters less if feedforward can be applied: compare the difference between curves (1) and (3) to that between curves (2) and (4) in Figure 7.

6 Conclusions

An explicite adaptive controller with disturbance measurement feedforward has been presented. It is based on polynomial LQG design and is capable of handling nonstationary and deterministic disturbances. The roles that the input penalty, the observer polynomial and feedforward control play in determining a compromise between ideal case performance and robustness have been exemplified. In simulation studies, the algorithm has been found to behave very well in general. One (seldomly occuring) remaining problem is that $\hat{A}$ and $\hat{B}$ may get unstable common factors when models are over-parametrized. The test of schemes such as that of De Laminat (1984) to avoid these situations is a focus of current research.

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