Cross-layer Optimization of Wireless Multi-hop Networks with Network Coding

Lei You[1,2] **Ping Wu[1]** **Zhiwen Pan[3]** **Honglin Hu[4]** **Junde Song[2]** **Mei Song[2]**

[1]Signals & Systems, Dept. of Engineering Sciences, Uppsala University, Uppsala, Sweden

[2]PCN&CAD Certer, Beijing University of Posts and Telecommunications, Beijing, China

[3]National Mobile Communication Research Laboratory, Southeast University, Nanjing, Jiangsu, China

[4]Shanghai Research Center for Wireless Communications, Shanghai, China

youlei.feng@gmail.com, Ping.Wu@angstrom.uu.se, pzw@seu.edu.cn, Honglin.hu@shrcwc.org, {songm, jdsong}@bupt.edu.cn

Abstract—**In this paper we present a cross-layer algorithm for joint optimization of congestion control, routing, and scheduling in wireless multi-hop networks with network coding. We introduce virtual flow variables in the formulation of capacity region of the networks. The utility maximization problem subject to constraints on the capacity region is solved using dual decomposition and subgradient method, based on which a new queuing model is obtained and which also can result in a cross-layer algorithm for distributed implementation. The new queuing model can facilitate coding operations and reduce coding complexity. Simulation results show that network coding in the proposed joint optimization algorithm can interact adaptively and optimally with other components in different layers, and thus yield higher performance than the routing scheme without network coding.**

Keywords-Wireless multi-hop network; Network coding; Cross-layer Optimization

I. INTRODUCTION

Network coding (NC) has emerged as a promising mechanism to efficiently utilize the resource of both wired and wireless networks. The pioneering work by Ahlswede *et al.* [1] has showed that maximum capacity of a multicast session can be achieved with NC. It is more attractive to implement NC in wireless networks due to the broadcast property of wireless medium, i.e., a single transmission from a node can be received by all its neighbours. A typical wireless NC is the opportunistic XOR coding, e.g., COPE scheme proposed in [2] that can identify coding opportunities and forward multiple packets (coded together) in a single transmission using broadcast advantage. However, integration of NC into the existing architecture of wireless network is not straightforward. Network coding can not work as an independent function in a specific layer. Optimal control of wireless multi-hop networks with NC involves interaction of NC with all the other functions in different layers (e.g., congestion control in transport layer, routing in network layer and scheduling in MAC layer).

Cross-layer optimization of wireless communication networks has been a very active research area in recent years. Backpressure-based approaches that determine routing and scheduling using queue backlog information is widely used for cross-layer design due to its optimality [14]. Joint optimization of congestion control, routing and scheduling for wireless multi-hop networks has been extensively studied using dual decomposition and sub-gradient method [3] [4]. Backpressure-based control algorithms for joint routing, scheduling and network coding across multiple uncast sessions are developed in [5] and [6] based on the poison-remedy approach [13]. The problem of coding-aware routing for wireless multi-hop networks is solved using linear programming, which is a centralized solution [7]. A backpressure-based optimal policy for joint scheduling and network coding with predetermined routing is proposed [8]. An oblivious backpressure algorithm for joint routing, scheduling and network coding is proposed in [9] for energy efficiency of wireless networks. However, optimal coding operation involves searching all the packets in all the queues, which results in a high complexity. In this paper, we consider joint optimization of congestion control, routing, scheduling and network coding for a wireless multi-hop network.

The paper is organized in the following way. In Section II the system model is discussed and the capacity region is formulated by introducing virtual flow variables. Then in Section III, the utility maximization problem is solved using dual decomposition and subgradient method, a new queuing model is obtained from the dual variables and then a cross-layer optimization algorithm is proposed. The simulation results are presented in Section IV, and the paper is concluded by Section V.

II. SYSTEM MODEL AND CAPACITY REGION WITH NETWORK CODING

A. System model

Consider a wireless multi-hop network, whose topology is represented by a directed graph $G=(N,L)$ with a node set N and a link set L. Link between nodes i and j is denoted by (i,j). The maximum transmission rate of each link (i,j) has a fixed value, c_{ij}. We assume that time is slotted and all the nodes in the network are synchronized.

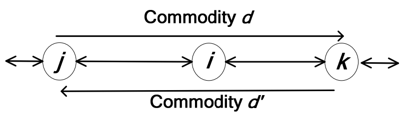

Fig.1 **OTIC** network coding

Denote x_i^d as the data rate of the session generated at node i and destined to node d. We consider a multi-commodity flow model and view packets to the same destination as one commodity. Let $D=\{d : d=1,\ldots,N\}$ be the set of commodities. It is easy to verify that it is inefficient to apply network coding to the sessions to a common destination. Here we consider one-hop network coding between local two-way commodities at intermediate nodes. Specifically, as showed in Fig. 1, assuming a commodity d traversing links (j,i) and (i,k), and

This work is sponsored by VINNOVA Sweden, CSC China, and the Ministry of Science and Technology China through National International Science and Technology Cooperation Project (NO. 2008DFA12090).

another commodity d' traversing links (k,i) and (i,j), then node i XORs the packets from d and d' and sends the coded packet through one broadcast to nodes j and k, where the coded packets will be decoded immediately using the packets that had traversed the node before. We refer to such a coding scheme as *One-hop Two-way Inter-Commodity* (**OTIC**) *network coding*.

We assume that each node uses a single half-duplex transceiver so that the node can not receive or transmit simultaneously. For OTIC network coding, a coded packet contains two packets that are from two different commodities (d and d') sent in reverse directions. Thus, there are two kinds of transmission: *unicast transmission* of native packets on a single link and *broadcast transmission* of coded packets on two links with a common transmitting node and two different receiving nodes. Denote (i,B) as a broadcast link with transmitting node i and receiving node set B which contains two nodes in the OTIC case, i.e., $B=\{j,k\}$ and $|B|=2$. In the following, we also use $(i,(j,k))$ to denote broadcast link (i,B). Transmission rate is denoted as $c_{i,(j,k)}$, which is defined as $c_{i,(j,k)}=\min(c_{ij},c_{ik})$. Let $(ij,(j,k))$ denote the broadcast transmission on link $(i,(j,k))$ and received by node j. Thus, we have $c_{ij,(j,k)}=c_{ik,(j,k)}=c_{i,(j,k)}$. Let E denote the set of all the possible broadcast links with each element denoted by e.

We consider K-hop interference model in which links within a K-hop range can not be active simultaneously. Using the method in [7], we can construct a *conflict graph* to identify all possible independent sets of links. An independent set is such a set of unicast and broadcast links that can be active simultaneously, because each is out of the K-hop range of other. Let Π denote the set of all the independent sets, and π an independent set that is a combined rate vector (r_u^π, r_B^π) with a $|L|+|E|$-dimension. The rate vector r_u^π is of $|L|$-dimension, and its l-th element can be written as

$$r_u^\pi(l)=\begin{cases} c_l & \text{if unicast link } l\in\pi \\ 0 & \text{otherwise}\end{cases}$$

and the rate vector r_B^π is of $|E|$-dimension and its e-th element can be expressed in the following manner

$$r_B^\pi(e)=\begin{cases} c_e & \text{if broadcast link } e\in\pi \\ 0 & \text{otherwise}\end{cases}$$

The feasible transmission rate region in the link layer using a time-sharing scheme is the convex hull of all the $|L|+|E|$-dimension rate vectors in Π, which can be expressed as

$$Co(\Pi)=\left\{(r_u,r_B):(r_u,r_B)=\sum_{\pi\in\Pi}\alpha_\pi(r_u^\pi,r_B^\pi),\ \alpha_\pi\ge 0,\ \sum_{\pi\in\Pi}\alpha_\pi=1\right\}$$

B. Cross-layer problem formulation

Let f_{ij}^d denote the native flow rate of commodity d on unicast link (i,j), and $f_{ij,(j,k)}^{d,(d')}$ the coding flow rate of commodity d that is coded with commodity d' at node i, broadcasted on link $(i,(j,k))$ and received by node j. Note that $f_{ij,(j,k)}^{d,(d')}$ and $f_{ik,(j,k)}^{d',(d)}$ are *virtual information rates* for transmission on link $(i,(j,k))$. To decode the coded packets correctly after the broadcast transmission, the previous node of packets of commodity d coded in virtual flow $f_{ij,(j,k)}^{d,(d')}$ is node k, while that of commodity d' in flow $f_{ik,(j,k)}^{d',(d)}$ is node j. Denote $P=\{(d,d'):\ d,d'\in D, d\ne d'\}$ as the set of all feasible commodity pairs. To develop a cross-layer algorithm with a low coding complexity, we formulate the capacity region of wireless multi-hop network with network coding by introducing virtual flow rate variables $\{g_{ji}^d\}$, where g_{ji}^d represents the total unicast rates (without coding) of packets at node i of commodity d that come from node j. With those virtual flows defined, the capacity region for a wireless multi-hop network with OTIC NC can be formulated as follows,

$$f_{ij}^d\ge 0,\ f_{ij,(j,k)}^{d,(d')}\ge 0,\ f_{dj}^d=f_{dj,(j,k)}^{d,(d')}=f_{ij,(j,k)}^{d,(d)}=0 \quad \forall i,j,k\in N,\ d,d'\in D \tag{1}$$

$$f_{ji}^d+\sum_{\substack{(k,j)\in L,\\ k\ne i}}\sum_{(d,d')\in P} f_{ji,(i,k)}^{d,(d')}\le g_{ji}^d+\sum_{\substack{(i,k)\in L,\\ k\ne j}}\sum_{(d,d')\in P} f_{ik,(j,k)}^{d,(d')} \quad \forall i\in N,\ d\in D,\ i\ne d,\ (j,i)\in L \tag{2}$$

$$x_i^d+\sum_{(j,i)\in L} g_{ji}^d\le \sum_{(i,j)\in L} f_{ij}^d \quad \forall i\in N,\ d\in D,\ i\ne d \tag{3}$$

$$\sum_{(j,i)\in L} g_{ji}^d\le V_i^d,\quad \forall i\in N,\ d\in D \tag{4}$$

$$g_{ji}^d\ge 0,\ g_{di}^d=g_{jd}^d=0\ \ \forall i,j\in N,\ d\in D \tag{5}$$

$$\sum_{d\in D} f_{ij}^d\le r_{ij},\ \forall(i,j),\ \sum_{(d,d')\in P} f_{ij,B}^{d,(d')}\le r_{i,B}, \forall(i,B),\ j\in B \tag{6}$$

$$\mathbf{r}=(r_u,r_B)\in Co(\Pi),\text{ where } r_u=(r_{ij}),\ r_B=(r_{i,B}) \tag{7}$$

where $V_i^d, \forall i\in N,\ d\in D$, are fixed and finite constants.

Constraints in (1) present that all the flow variables are nonnegative, all the flows can not be sent to other nodes by their destinations, and the packets of the same commodity can not be coded together; Constraints in (2) and (3) are the flow conservation law for each commodity at nodes that are not its destination. (6) defines the constraints on link capacity allocation. (7) is the constraint on link rate region of the wireless multi-hop network with OTIC network coding with time-sharing; (4) represents the constraints on the virtual flow rates of each commodity at each node. Since the virtual flow variables are added artificially, each V_i^d should be set to be sufficiently large (e.g., larger than the sum of the input rates of the corresponding node) so that the capacity region is not smaller than that without virtual flows.

The objective of our cross-layer optimization problem is to achieve fairness among all the sessions by maximizing the sum of utilities of all the sessions subject to the constraints (1)-(7) on the capacity region as follow,

$$\text{Maximize } \sum_{i,d\in N} U_i^d(x_i^d) \qquad \textbf{(P1)}$$
$$\text{s.t. (1)-(7)}$$

where the utility function $U_i(x_i)$ associated with session i with sending rate x_i is assumed to be a concave, non-decreasing and continuously differentiable function. Maximizing the total utility can achieve proportional fairness if $U(z)=\log(z)$.

Primal problem **P1** is a convex problem with linear constraints and a concave objective function [10]. It can be solved using interior-point method. But this requires centralized computation. We will show that one can use dual decomposition and subgradient method [3] [4] to **P1** to find a cross-layer optimization solution which is possibly implemented in a distributed manner, and to obtain a new queuing model that can facilitate coding operation as well.

III. Cross-layer Optimization Algorithm

Since **P1** is convex, strong duality holds. Thus, it can be solved optimally by solving its dual problem. By introducing Lagrange multipliers $\{p_{ji}^d\}$ and $\{q_i^d\}$ to relax constraints in (2) and (3), respectively, the corresponding partial Lagrangian of **P1** can be written as follows

$$\begin{aligned} &L(\mathbf{x},\mathbf{f},\mathbf{f_{nc}},\mathbf{g};\mathbf{p},\mathbf{q}) \\ &= \sum_{i,d\in N} U_i^d(x_i^d) \\ &+ \sum_{i\in N}\sum_{(j,i)\in L}\sum_{d\in D} p_{ji}^d [\, g_{ji}^d + \sum_{\substack{(i,k)\in L,\\ k\neq j}}\sum_{(d,d')\in P} f_{ik,(j,k)}^{d,(d')} - f_{ji}^d - \sum_{\substack{(k,j)\in L,\\ k\neq i}}\sum_{(d,d')\in P} f_{ji,(i,k)}^{d,(d')} \,] \\ &+ \sum_{i\in N}\sum_{d\in D} q_i^d [\sum_{(i,j)\in L} f_{ij}^d - x_i^d - \sum_{(j,i)\in L} g_{ji}^d] \end{aligned}$$

where $\mathbf{x},\mathbf{f},\mathbf{f_{nc}}$ and $\mathbf{g}$ represent the vectors of primal variables of session rates, native flow rates, coding flow rates and virtual flow rates, respectively; $\mathbf{p}$ and $\mathbf{q}$ represent the vectors of corresponding dual variables. With this Langrangian we can define the dual objective function as

$$D(\mathbf{p},\mathbf{q}) = \begin{cases} \max_{\mathbf{x},\mathbf{f},\mathbf{f_{nc}},\mathbf{g}} L(\mathbf{x},\mathbf{f},\mathbf{f_{nc}},\mathbf{g};\mathbf{p},\mathbf{q}) \\ s.t. \quad (1) \text{ and } (4)\text{-}(7) \end{cases} \tag{8}$$

Thus, the dual problem of problem **P1** can be written as:

$$\begin{aligned} &\text{Minimize } D(\mathbf{p},\mathbf{q}) \\ &s.t.\ \mathbf{p}\geq 0,\ \mathbf{q}\geq 0 \end{aligned} \qquad (\mathbf{D1})$$

which is also a convex problem [10].

A. Subgradient method to solve *D1*

Subgradient method is effective to solve dual problem **D1** since its objective function (8) is not differentiable [11]. Supposing primal variables $\mathbf{x},\mathbf{f},\mathbf{f_{nc}}$ and $\mathbf{g}$ are obtained as a solution of maximization in dual objective function (8) at point $(\mathbf{p},\mathbf{q})$, the subgradients of function (8) at p_{ji}^d and q_i^d can be expressed as follows, respectively,

$$\begin{cases} Gr_{ji}^d = g_{ji}^d + \sum_{\substack{(i,k)\in L,\\ k\neq j}}\sum_{(d,d')\in P} f_{ik,(j,k)}^{d,(d')} - f_{ji}^d - \sum_{\substack{(k,j)\in L,\\ k\neq i}}\sum_{(d,d')\in P} f_{ji,(i,k)}^{d,(d')} \\ Gr_i^d = \sum_{(i,j)\in L} f_{ij}^d - x_i^d - \sum_{(j,i)\in L} g_{ji}^d \end{cases} \tag{9}$$

Subgradient method finds the optimal solution of **D1** by updating dual variables in each iteration step t using subgradients in (9) in the following way until the solution converges.

$$\begin{cases} p_{ji}^d(t+1) = \left[p_{ji}^d(t) - \alpha(t)\,(Gr_{ji}^d)\right]^+ \\ q_i^d(t+1) = \left[q_i^d(t) - \alpha(t)\,(Gr_i^d)\right]^+ \end{cases} \tag{10}$$

where $\alpha(t)$ is a positive step-size at step t and $[y]^+ = \max(y,0)$.

Subgradient method converges to the optimum of **D1** if $\alpha(t)$ is designed appropriately according to the rules in [11]. Since strong duality holds for **P1**, the primal variables $(\mathbf{x}^*,\mathbf{f}^*,\mathbf{f}_{\mathbf{nc}}^*,\mathbf{g}^*)$ related to the corresponding optimal dual variables $(\mathbf{p}^*,\mathbf{q}^*)$ are a globally optimal solution to **P1** [10].

B. Calculating subgradient by dual decomposition

To calculate subgradients (9) in each step t, we need to determine primal variables $\mathbf{x},\mathbf{f},\mathbf{f_{nc}}$ and $\mathbf{g}$, which is the solution of maximization problem in (8) at point $(\mathbf{p}(t),\mathbf{q}(t))$. The maximization problem in (8) can be decomposed into the following three subproblems:

$$D(\mathbf{p},\mathbf{q}) = D_1(\mathbf{q}) + D_2(\mathbf{p},\mathbf{q}) + D_3(\mathbf{p},\mathbf{q})$$

where

$$D_1(\mathbf{q}) = \max_{\mathbf{x}\geq 0} \sum_{i,d\in N} [U_i^d(x_i^d) - q_i^d x_i^d] \tag{11}$$

$$D_2(\mathbf{p},\mathbf{q}) = \begin{cases} \max_{\mathbf{g}} \sum_{i\in N}\sum_{(j,i)\in L}\sum_{d\in D} g_{ji}^d (p_{ji}^d - q_i^d) \\ s.t. \quad (4)\ (5) \end{cases} \tag{12}$$

$$D_3(\mathbf{p},\mathbf{q}) = \begin{cases} \max_{\mathbf{f},\mathbf{f_{nc}}} \begin{Bmatrix} \sum_{i\in N}\sum_{(i,j)\in L}\sum_{d\in D} f_{ij}^d (q_i^d - p_{ij}^d) + \\ \sum_{i\in N}\sum_{(i,j)\in L}\sum_{\substack{(i,k)\in L\\ k\neq j}}\sum_{(d,d')\in P} f_{i,(j,k)}^{d,(d')}[(p_{ji}^d - p_{ik}^d) + (p_{ki}^{d'} - p_{ij}^{d'})] \end{Bmatrix} \\ s.t. \quad (1), (6) \text{ and } (7) \end{cases} \tag{13}$$

Subproblems (11), (12) and (13) can be solved separately and the solutions actually result in a cross-layer optimization algorithm for joint congestion control, routing, scheduling and network coding, to be showed in Section III.**D.** Next, we first present the queuing model implied by dual variables $\mathbf{p}$ and $\mathbf{q}$.

C. Queuing model and evolution of the queues

If the step-size for updating dual variables is constant in each step, the variables are updated in a similar manner to the corresponding queues [3] [4]. For a sufficiently small, constant step-size, subgradient method can converge to a small neighbourhood of the optimal solution [11][4]. Similarly, assuming a constant step-size in (10), i.e., $\alpha(t)=\alpha$, we can associate each of the dual variables in $(\mathbf{p},\mathbf{q})$ with a queue, which results in a new queuing model at each node. Different from previous work [3] [4] where no network coding was considered and each node maintains only one queue for each commodity, each node in the present case maintains several queues for each commodity *d*, including single ***unicast queue***, Q_i^d, buffering packets to be sent out as native packets, and ***receiving queues*** $P_{ji}^d,\ \forall (j,i)\in L$, where each queue P_{ji}^d is used to buffer all the packets from neighbouring node *j*. There is a virtual link between Q_i^d and between each queue P_{ji}^d, respectively. The packets of commodity *d* that comes from neighbour *j* but are to be sent out as native packets (without coding) are transferred from P_{ji}^d to Q_i^d through the virtual link with virtual flow rate g_{ji}^d. Thus, the evolution of the queues in this queuing model is

$$\begin{cases} P_{ji}^d(t+1) = \left[P_{ji}^d(t) - (g_{ji}^d + \sum_{\substack{(i,k)\in L,\\ k\neq j}}\sum_{(d,d')\in P} f_{ik,(j,k)}^{d,(d')} - f_{ji}^d - \sum_{\substack{(k,j)\in L,\\ k\neq i}}\sum_{(d,d')\in P} f_{ji,(i,k)}^{d,(d')})\right]^+ \\ Q_i^d(t+1) = \left[Q_i^d(t) - (\sum_{(i,j)\in L} f_{ij}^d - x_i^d - \sum_{(j,i)\in L} g_{ji}^d)\right]^+ \end{cases} \tag{15}$$

Comparing (15) with (10) for positive constant $\alpha(t)=\alpha$, we can find the relationships of queue lengths with corresponding dual variables, $P_{ji}^d(t)=p_{ji}^d(t)/\alpha$ and $Q_i^d(t)=q_i^d(t)/\alpha$.

D. Cross-layer optimization algorithm

With the above queuing model, we can use queue length information instead of dual variables in the dual decomposition and subgradient method.

Algorithm 1: Cross-layer optimization algorithm for wireless multi-hop network with OTIC NC

Assuming that the lengths of unicast and receiving queues are $Q_i^d(t)$ and $P_{ji}^d(t)$, $\forall i\in N, d\in D, (j,i)\in L$, the network performs the following operations at slot *t*:

1) <u>*Congestion control:*</u> Each session x_i^d calculates its rate using the length of local unicast queue $Q_i^d(t)$ at node *i* as $x_i^d=\max(0, \mathrm{U}_i^{d\prime-1}[\mathrm{Q}_i^d(t)/\alpha])$, where $\mathrm{U}_i^{d\prime-1}(\bullet)$ is the inverse function of derivative of utility function $\mathrm{U}_i^d(\bullet)$, α is the predetermined constant parameter (step-size).

2) <u>*Unicast queue loading:*</u> Each node *i*

- puts the packets generated from session $x_i^d(t)$ into unicast queue Q_i^d, and
- finds $j^*=\arg\max_{(j,i)\in L}(P_{ji}^d(t)-Q_i^d(t))$ for each commodity *d*, then sets $g_{ji}^d(t)=\begin{cases}V_i^d & \text{if } j=j^* \text{ and } P_{j^*i}^d(t)-Q_i^d(t)>0\\ 0 & \text{otherwise}\end{cases}$ and transfers packets from queue $P_{ji}^d(t)$ to queue $Q_i^d(t)$ at rate $g_{ji}^d(t)$.

3) <u>*Link Scheduling*</u>*:* Each node *i*

- finds $d^*=\arg\max_{d\in D}(Q_i^d(t)-P_{ij}^d(t))$ and then calculates a weight $w_{ij}=\max(0,\ Q_i^{d^*}(t)-P_{ij}^{d^*}(t))$ for each unicast link (*i*, *j*);
- finds $(d^*,d'^*)=\underset{\substack{(d,d')\in P,\,(P_{ji}^d-P_{ik}^d)>0,\\ (P_{ki}^{d'}-P_{ij}^{d'})>0}}{\arg\max}[(P_{ji}^d-P_{ik}^d)+(P_{ki}^{d'}-P_{ij}^{d'})]$ and then calculates $w_{i,(j,k)}=(P_{ji}^{d^*}(t)-P_{ik}^{d^*}(t))+(P_{ki}^{d'^*}(t)-P_{ij}^{d'^*}(t))$ for each broadcast link (*i*, (*j*, *k*)).
- Some scheduling algorithm picks an optimal link independent set π^* that satisfies the relation

$$\pi^*\in\max_{\pi\in\Pi}\left\{\sum_{i\in N}\sum_{(i,j)\in L}r_u^\pi(i,j)w_{ij}+\sum_{i\in N}\sum_{(i,j)\in L}\sum_{\substack{(i,k)\in L\\ k\neq j}}r_B^\pi(i,(j,k))w_{i,(j,k)}\right\} \quad (16)$$

4) <u>*Network coding and Routing:*</u> Each node *i*

- XORs packets from receiving queues $P_{ji}^{d^*}$ and $P_{ki}^{d'^*}$ respectively, and broadcasts the coded packets on links (i,j) and (i,k) at rate $\min(c_{ij},c_{ik})$, if broadcast link $(i,(j,k))\in\pi^*$ and (d^*,d'^*) are the commodity pair that attains $w_{i,(j,k)}$.
- transmits the packets from unicast queue $Q_i^{d^*}$ on link (i,j) at rate c_{ij} if unicast link $(i,j)\in\pi^*$ and d^* are the commodity pair that attains w_{ij}.

5) <u>*Queue information exchanging:*</u>

<u>At the end of this slot, all the queues are updated according</u> to (15). All the nodes exchange their queue length information with their neighbours through a certain control channel.

From step 4) in the algorithm, we can see that the developed queuing model facilitates the optimal coding operation. Different from one queue per one commodity model, it neither has to maintain for each packet the set of nodes which it comes from nor has to search all the queues for packets that can be coded together in each slot. For network coding, a node just needs to pick packets from corresponding receiving queues. Thus the complexity of finding valid coding opportunity is reduced.

E. Link scheduling algorithm

Note that **Algorithm 1** is possible to be implemented in a distributed way except for link scheduling (16) which is a maximum weighted independent set problem that is NP-hard and needs a centralized solution under general *K*-hop interference model.

Since in this paper we are mainly interested in the optimal interaction of functions at different layers with network coding, we test the proposed cross-layer optimization algorithm using a centralized *Greedy Maximal Scheduling* (GMS) algorithm [12] for link scheduling (16) in the simulation in the next section. GMS each time schedules link with the highest weight (w_{ij} or $w_{i,(j,k)}$) in (16) and drops the links in the interfering range of the selected link until all the links in the network are either scheduled or dropped. However, the design and performance analysis of the distributed link scheduling algorithm with low complexity under more general interference model are to be done in our future work.

IV. SIMULATIONS RESULTS

In this section we evaluate the performance of the proposed cross-layer optimization algorithm **t**hrough simulations. The topology and link rates used herein are showed in Fig. 2. Note that we set $c_{21}=c_{32}=c_{42}=c_{64}=0$ to get an acyclic topology. There exist two sessions: x_1^9 from node 1 to node 9, and x_9^3 from node 9 to node 3. We set the step-size $\alpha=0.01$ and use utility function $U(x)=\ln(x)$. We assume primary interference models in which links share a common transmitting node and receiving nodes can not be active simultaneously, and we use the greedy maximal scheduling (GMS) algorithm as stated in Section III.**E** for all the simulations.

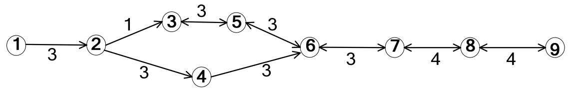

Fig.2 Topology used in the simulations.

A. Comparison

Comparison of **Algorithm 1** is made in terms of converged average session rate, with the cross-layer algorithm based on a pure routing scheme without network coding developed in [10]. As shown in Fig. 3, we see that from **Algorithm 1** the converged average rates of flows 9-1 and 3-9 are $x_1^9=0.9231$ and $x_9^3=0.8451$, respectively, while from the algorithm without network coding the converged average rates are $x_1^9 = 0.75074$ and $x_9^3 = 0.7497$, respectively. This shows that the performance of the cross-layer optimization with NC is improved by up to 17.88% in this scenario.

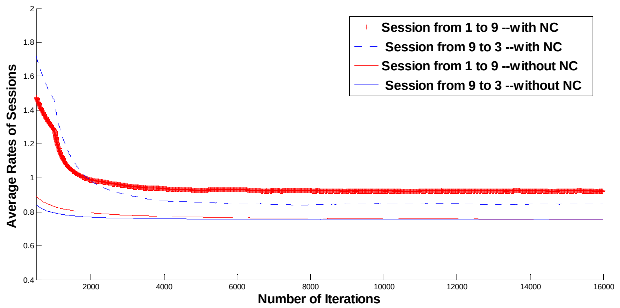

Fig.3 Converged average rates with NC and without NC

B. Allocation of flow rates on all the links

Tables 1 and 2 show the average flow rate allocations on each link for cross-layer optimization algorithms without network coding (in [10]) and with network coding (**Algorithm 1**), respectively. From the tables it follows three observations:

(1) Without network coding, flow x_1^9 only follows path 1->2->4->6->9 to avoid interference with flow x_9^3, while with network coding, part of flow x_1^9 goes along path 1->2->3->5->6->9 to create coding opportunity with x_9^3 at node 6. This is because network coding can increase the utilization of node 6 (which is a bottleneck) by broadcasting the coded packets. This shows that **Algorithm 1** can find the coding aware multipath routing in an optimal and adaptive way.

(2) Nodes 5 and 6 take all coding opportunities for the two flows to increase the utilization of wireless medium through broadcast advantage. The average coded flow rates on links (5, 3), (5, 6), (6, 5) and (6, 7) are about 0.54 (if we ignore the imprecision of computation in the simulation).

(3) Only part of the two flows meeting at nodes 7 and 8 are coded together. This is because the nodes are less congested than nodes 5 and 6, and thus it is no use of applying coding all the time. This can reduce the complexity of coding and decoding operation.

These observations show that **Algorithm 1** can determine necessary network coding and perform joint congestion control, routing, scheduling and network coding in an adaptive and optimal manner.

V. CONCLUSION

In this paper, we proposed a cross-layer algorithm for joint optimization of congestion control, routing, scheduling and network coding for a wireless multi-hop network with network coding. Virtual flow variables are introduced to formulate the capacity region. After solving the utility maximization problem subject to constraints on the capacity region using dual decomposition and subgradient method, we obtain a cross-layer optimization algorithm with a new queuing model that can reduce the complexity of the coding operation. Simulations have showed that, with the proposed algorithm and queuing model, network coding can interact adaptively and optimally with the other components in the network layer.

REFERENCES

[1] R. Ahlswede, N. Cai, S.-Y. Li, and R. Yeung. Network information flow. IEEE Trans. Info. Theory, 46(4):1204–1216, Jul. 2000.

[2] S. Katti, H. Rahul, W. Hu, D. Katabi, M. M´edard, and J. Crowcroft, “XORs in the air: Practical wireless network coding,” in Proc. of ACM SIGCOMM, Oct. 2006, pp. 243–254.

[3] X. Lin and N. Shroff, Joint rate control and scheduling in multihop wireless networks, 43th IEEE CDC, pp.1484-1489, Vol.2, 2004..

[4] L. Chen, S. H. Low, M. Chiang, and J. C. Doyle, “Cross-layer congestion control, routing and scheduling design in ad hoc wireless networks,” in Proc. of IEEE Infocom, Apr. 2006.

[5] T. Ho, Y. Chang, and K. J. Han, “On constructive network coding for multiple unicasts,” in Proc. of Allerton Conf. on Comm., Contr. Comput., Sept. 2006.

[6] A. Eryilmaz and D. S. Lun, “Control for inter-session network coding,” in Proc. of Workshop on Network Coding, Theory & Applications, 2007.

[7] S. Sengupta, S. Rayanchu, and S. Banerjee, “An analysis of wireless network coding for unicast sessions: The case for coding-aware routing,” in Proc. of IEEE Infocom, May 2007, pp. 1028–1036.

[8] P. Chaporkar, A. Proutiere. Adaptive network coding and scheduling for maximizing throughput in wireless networks. in Proc. of MobiCom’07.

[9] Tao Cui, Lijun Chen, Ho, T., “ Energy Efficient Opportunistic Network Coding for Wireless Networks,” in Proc. of IEEE INFOCOM 2008.

[10] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[11] D. P. Bertsekas, Nonlinear Programming, 2nd ed. Belmont, MA: Athena Scientific, 1999.

[12] X. Lin and N. Shroff, “The impact of imperfect scheduling on cross-layer congestion control in wireless networks,” IEEE/ACM Trans. Networking, vol. 14, no. 2, pp. 302–315, April 2006.

[13] D. Traskov, N. Ratnakar, D. S. Lun, R. Koetter, and M. M´edard, “Network coding for multiple unicasts: An approach based on linear optimization,” in Proc. of IEEE ISIT, July 2006, pp. 1758–1762

[14] L. Tassiulas and A. F. Ephremides. , “Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks, ” IEEE Transactions on Automatic Control, Dec.1992,37(12):1936–1948, .

Table.1. Allocation of average flow rates on each link without network coding

Link		(1,2)	(2,4)	(4,6)	(2,3)	(3,5)	(5,6)	(6,7)	(7,8)	(8,9)
Flow 9-1	flow rate on each link	0.7507	**0.7507**	0.7507	0	0	0	0.7507	0.7507	0.7507
Link		**(9,8)**	**(8,7)**	**(7,6)**	**(6,5)**	**(5,3)**	--------	-------	-------	------
Flow 3-9	flow rate on each link	0.7497	0.7497	0.7497	0.7497	0.7497				

Table.2. Allocation of average flow rates on each link with network coding

Link		(1,2)	(2,4)	(4,6)	(2,3)	(3,5)	(5,6)	(6,7)	(7,8)	(8,9)
Flow 9-1	Coded flow rate	0	0	0	0	0	0.5335	0.5410	0.5455	0.3257
	Uncoded flow rate	0.9231	**0.3866**	0.3866	**0.5380**	0.5385	0.0075	0.3821	0.3776	0.5974
Link		**(9,8)**	**(8,7)**	**(7,6)**	**(6,5)**	**(5,3)**	--------	----------	--------	-------
Flow 3-9	Coded flow rate	0	0.3257	0.5455	0.5410	0.5335				
	Uncoded flow rate	0.8451	0.5215	0.2997	0.3042	0.3117				