

# Effect of Channel Prediction Errors on Adaptive Modulation Systems for Wireless Channels

Sorour Falahati<sup>1</sup>, Arne Svensson<sup>1</sup>, Torbjörn Ekman<sup>2</sup> and Mikael Sternad<sup>2</sup>

<sup>1</sup>Department of Signals and Systems, Chalmers University of Technology, SE-412 96 Göteborg, Sweden.

Email: {Sorour.Falahati, Arne.Svensson}@s2.chalmers.se

<sup>2</sup>Signals and Systems, Department of Material Science, Uppsala University, SE-751 20 Uppsala, Sweden.

Email: {Torbjorn.Ekman, Mikael.Sternad}@signal.uu.se

## Abstract

We study the optimum design of an adaptive modulation scheme based on M-QAM modulation assisted by channel prediction for the flat Rayleigh fading channel. The data rate and transmit power are adapted to maximize the spectral efficiency, subject to average power and instantaneous BER constraints. Optimum solutions for the rate and transmit power are derived based on the predicted SNR as well as the prediction error variance which are assumed to be available at the transmitter. The analytical results are evaluated and presented.

## 1 Introduction

Adaptive modulation, or link adaptation, is a powerful technique for improving the spectral efficiency in wireless transmission over fading channels. The modulation parameters, such as signal constellation size, transmitted power level and data rate are here adjusted according to the channel conditions. The adaptation can also take requirements of different traffic classes and services such as required bit error rates, into account. In the case of *fast* link adaptation considered here, we strive to adapt to the small scale fading. The receiver estimates the received power and sends feedback information via a return channel to the transmitter, which adjusts its modulation parameters. Due to the unavoidable delays involved in power estimation, feedback transmission and modulation adjustment, the adaptation needs to be based on *predicted* estimates of the power of the fading communication channel.

The optimum design of adaptive modulation systems is extensively studied and thoroughly covered in the literature (e.g. see [1], [2], [3]). However, in the proposed solutions, perfect knowledge of the channel conditions at the transmitter as well as error free channel estimates at the receiver are common assumptions for the system design and performance

evaluation. In real systems, these assumptions are not valid. This leads to performance degradation such as decrease in the throughput and increase in the delay as well as failure in providing the required service such as the expected bit error rate. Therefore, a solution based on a more realistic assumption is of great importance and interest.

In this paper, we intend to design an optimum adaptive modulation system for a given channel prediction error variance with a corresponding statistical model, which maximizes the spectral efficiency while satisfying a BER requirement. The proposed system utilizes a linear filter to predict the channel quality at the receiver [4], [5]. Moreover a statistical model which reasonably well describes the properties of prediction errors is investigated. The model gives a good estimate of the mean, variance and probability density function of the errors. Then, the statistical model is taken into account for performance analysis of the adaptive modulation schemes such as for example, the Bit Error Rate (BER) versus Signal-to-Noise Ratio (SNR) for given prediction error variances.

This paper is organized as follows. Section 2 describes the system model and the notations which are used throughout this study. The channel prediction is explained in Section 3. The BER formula as a function of predicted instantaneous SNR is evaluated in Section 4 and the optimal rate and power adaptation are derived in Section 5. Analytical results are presented in Section 6 and finally, some conclusions are drawn in Section 7.

## 2 System Model

In the adaptive modulation scheme, M-QAM modulation schemes with different constellation sizes are provided at the transmitter. For each transmission, the modulation scheme and transmit power are chosen to maximize the spectral efficiency for the instantaneous BER and average power constraints based on the instantaneous predicted SNR. The channel is modeled

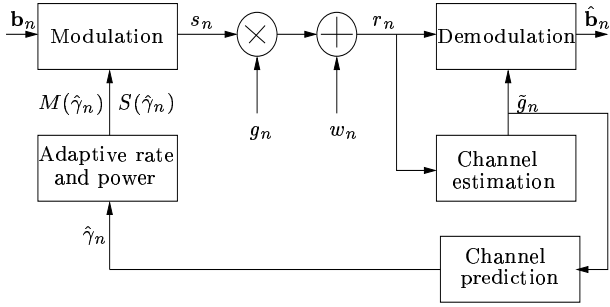


Figure 1: Discrete model of the system.

by a flat Rayleigh fading. At the receiver, demodulation is performed using the channel estimation. The discrete model of the system is depicted in Figure 1. All the signals are sampled at the symbol rate where the index  $n$  represents the signal sample at time  $nT_s$  where  $T_s$  is the symbol period.  $g_n$  is the complex channel gain with uniformly distributed phase within  $[0, 2\pi)$  and Rayleigh distributed amplitude with pdf (probability density function) given by

$$f_{|g|}(|g_n|) = \frac{|g_n|}{\sigma_g^2/2} \exp\left(-\frac{|g_n|^2}{\sigma_g^2}\right) \quad (1)$$

where  $\sigma_g^2$  is the average power of the fading process. Moreover, the auto-correlation function of the complex channel gain is given by

$$r_g(m) = E(g_n g_{n-m}^*) = \sigma_g^2 J_0(2\pi f_d T_s m) \quad (2)$$

where  $E(\cdot)$  is the Expectation function, the asterisk denotes the conjugate operator,  $J_0(\cdot)$  is the Bessel function of order zero and  $f_d$  is the maximum Doppler frequency<sup>1</sup>.  $w_n$  is a sample of the complex AWGN with zero mean and time-invariant variance  $\sigma_w^2$ .  $y_n$  is the noisy observation of  $g_n$  at the receiver which is used by an FIR filter at the receiver to predict the instantaneous received SNR denoted by  $\hat{\gamma}_n$  where an error free feed-back channel is assumed. Furthermore, the feed-back transmission delay is presumed to be taken up by the predictor since the prediction is performed beyond the corresponding delay.

Based on  $\hat{\gamma}_n$ , the modulation scheme with constellation size  $M(\hat{\gamma}_n)$  (out of  $N$  constellations available at the transmitter) which transmits  $k(\hat{\gamma}_n) = \log_2 M(\hat{\gamma}_n)$  bits per symbol, and the transmit power  $S(\hat{\gamma}_n)$  are selected. Each block of  $k(\hat{\gamma}_n)$  data bits denoted by  $\mathbf{b}_n$ , is Gray encoded and mapped to  $s_n$  which is a symbol in the signal constellation and is transmitted over the flat Rayleigh fading channel. The received sample,  $r_n$ , is used to estimate the channel gain  $\tilde{g}_n$  which is used to demodulate  $r_n$  to detect the transmitted bits denoted by  $\hat{\mathbf{b}}_n$ . Since the estimation error is believed

<sup>1</sup>In the following  $r_g$  is equivalently used instead of  $r_g(0)$ .

to have a minor effect on the performance compared to the prediction error, perfect channel estimation is assumed in the demodulation here, i.e.  $\tilde{g}_n = g_n$ . In future extension of this work, we however hope to be able to include estimation error to the model. The SNR prediction  $\hat{\gamma}_n$  is here assumed to be based on noisy channel estimates  $\tilde{g}_n$  and will therefore have a prediction error, with variance  $\sigma_\epsilon^2$ .

In this study, the following notations similar to [1] are used. Let  $\bar{S}$  denote the average transmit signal power and  $\bar{\gamma} = \bar{S}/\sigma_w^2$  denote the average received SNR for the channel gain with normalized average power ( $\sigma_g^2 = 1$ ). For a constant transmit power  $\bar{S}$ , the instantaneous received SNR is  $\gamma_n = \bar{\gamma} p_n$  where  $p_n = |g_n|^2$  is the instantaneous channel power gain. Accordingly, the instantaneous predicted received SNR is  $\hat{\gamma}_n = \bar{\gamma} \hat{p}_n$  where  $\hat{p}_n = |\hat{g}_n|^2$  is the predicted instantaneous channel power gain. For the transmit power  $S(\hat{\gamma}_n)$ , the instantaneous received SNR is given by  $\gamma_n(S(\hat{\gamma}_n)/\bar{S})$ . Also, the rate region boundaries are denoted by  $\{\hat{\gamma}_i\}_{i=0}^{N-1}$ , defined as the ranges of  $\hat{\gamma}_n$  values over which the different constellations are used by the transmitter, where  $\hat{\gamma}_N = \infty$ . When the predicted instantaneous SNR belongs to a given rate region, i.e.  $\hat{\gamma}_n \in [\hat{\gamma}_i, \hat{\gamma}_{i+1})$ , the corresponding constellation of size  $M(\hat{\gamma}_n) = M_i$  with  $k(\hat{\gamma}_n) = k_i$  bits per symbol is transmitted. Finally, there is no transmission if  $\hat{\gamma}_n < \hat{\gamma}_0$  meaning that  $\hat{\gamma}_0$  is the cutoff SNR.

### 3 Channel Prediction

The channel gain in a Rayleigh fading mobile radio channel takes values from a complex time series which can be modeled as a correlated complex Gaussian stochastic process. The absolute square, i.e. the power, of the time series is predicted based on linear regression of the observations of the complex time series. The unbiased quadratic predictor that is optimal in the Mean Square Error (MSE) sense is derived in [4] where it is shown that the same prediction coefficients that are optimal for the linear prediction of the complex tap, are optimal for the quadratic unbiased predictor.

The complex channel gain is observed in noise as

$$y_n = g_n + e_n \quad (3)$$

where  $e_n$  is assumed to be a complex Gaussian random variable which is independent from  $g_n$ , has zero mean and auto-correlation given by

$$r_e(m) = E(e_n e_{n-m}^*) = \sigma_e^2 \delta(m). \quad (4)$$

where  $\delta(\cdot)$  is the Kronecker delta function.

From the past and present observations, the power of the signal at time  $n + L$ , i.e.  $p_{n+L} = |g_{n+L}|^2$ , is predicted by an unbiased FIR predictor. In the vector

formulation, the unbiased predicted power based on the past noisy observations is given by

$$\hat{p}_{(n+L|n)} = \theta^H \varphi_n \varphi_n^H \theta + r_g - \theta^H \mathbf{R}_\varphi \theta \quad (5)$$

where

$$\varphi_n = [y_n, y_{n-1}, \dots, y_{n-(K-1)}]^H, \quad (6)$$

is the regressor,  $K$  is the number of filter coefficients,  $\theta$  is a column vector containing the complex filter coefficients,  $H$  is the Hermitian operator and  $\mathbf{R}_\varphi$  is the  $K \times K$  correlation matrix for the regressors.  $\mathbf{R}_\varphi$  is given by  $\mathbf{R}_\varphi = \mathbf{R}_g + \mathbf{R}_e$  where

$$\mathbf{R}_g(n, m) = r_g(|n - m|) \quad \text{for } n, m = 1, \dots, K \quad (7)$$

and

$$\mathbf{R}_e = \sigma_e^2 \mathbf{I}_{K \times K} \quad (8)$$

where  $\mathbf{I}$  is the identity matrix. Moreover, (5) corresponds to the absolute square of the complex prediction, but with a bias compensation added to it to provide an unbiased power estimate, i.e.

$$\hat{p}_{(n+L|n)} = |\hat{g}_{(n+L|n)}|^2 + r_g - \theta^H \mathbf{R}_\varphi \theta. \quad (9)$$

The minimum MSEs of the channel gain prediction error (i.e.  $\epsilon_{c_n} = g_n - \hat{g}_{(n|n-L)}$ ) and the power prediction error (i.e.  $\epsilon_{p_n} = p_n - \hat{p}_{(n|n-L)}$ ) are given by

$$\sigma_{\epsilon_c}^2 = r_g - \mathbf{r}_{g\varphi}^H \mathbf{R}_\varphi^{-1} \mathbf{r}_{g\varphi} \quad (10)$$

and

$$\sigma_{\epsilon_p}^2 = r_g^2 - |\mathbf{r}_{g\varphi}^H \mathbf{R}_\varphi^{-1} \mathbf{r}_{g\varphi}|^2, \quad (11)$$

respectively, where  $\mathbf{r}_{g\varphi}$  is the cross-correlation between the signal and the regressor given by

$$\begin{aligned} \mathbf{r}_{g\varphi} &= \mathbb{E}\{g_n \varphi_{n-L}\} \\ &= [r_g(L), r_g(L+1), \dots, r_g(L+(K-1))]^T. \end{aligned} \quad (12)$$

Moreover, the filter coefficient vector  $\theta$ , that minimizes the corresponding error variances is given by

$$\theta = \mathbf{R}_\varphi^{-1} \mathbf{r}_{g\varphi}. \quad (13)$$

Also (10) and (11) are related through

$$\sigma_{\epsilon_p}^2 = \sigma_{\epsilon_c}^2 (2r_g - \sigma_{\epsilon_c}^2). \quad (14)$$

In order to solve the optimization problem, the pdf of the instantaneous SNR is required. Since  $g_n$  is a complex Gaussian random variable, both the true and predicted powers, i.e.  $p_n = |g_n|^2$  and  $\hat{p}_n = |\hat{g}_n|^2$  are  $\chi^2$  distributed. In [5], it is shown that the pdf of  $\gamma$  conditioned on  $\hat{\gamma}$  is given by

$$\begin{aligned} f_\gamma(\gamma|\hat{\gamma}) &= \frac{U(\gamma)U(\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2)}{\bar{\gamma}\sigma_{\epsilon_c}^2} \exp\left(-\frac{\gamma + \hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2}{\bar{\gamma}\sigma_{\epsilon_c}^2}\right) \times \\ &I_0\left(\frac{2}{\bar{\gamma}\sigma_{\epsilon_c}^2} \sqrt{\gamma(\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2)}\right), \end{aligned} \quad (15)$$

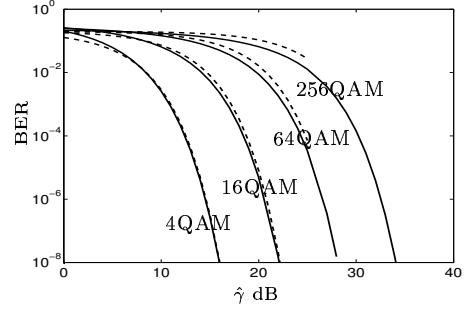


Figure 2: BER versus instantaneous predicted SNR of M-QAM schemes for  $\hat{\gamma} = 30$  dB and  $\sigma_{\epsilon_p}^2 = 0.001$ . The solid lines and dashed lines correspond to (19) based on (17) and (18), respectively.

where  $U(\cdot)$  is the Heaviside's step function and  $I_0(\cdot)$  is the zeroth order modified Bessel function. The pdf of  $\hat{\gamma}$  is given by

$$f_{\hat{\gamma}}(\hat{\gamma}) = \frac{U(\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2)}{\bar{\gamma}(r_g - \sigma_{\epsilon_c}^2)} \exp\left(-\frac{\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2}{\bar{\gamma}(r_g - \sigma_{\epsilon_c}^2)}\right). \quad (16)$$

The index  $n$  is dropped in the pdf expressions since  $g_n$  and  $\hat{g}_n$  are both stationary random processes.

## 4 M-QAM BER Performance

The transmitter adjusts the constellation size and the transmit power based on the instantaneous predicted SNR  $\hat{\gamma}$ . Evaluation of the optimal power and constellation size (or rate) which maximize the spectral efficiency and satisfy the BER requirement, requires the BER formula as a function of  $\hat{\gamma}$ . Assuming a square M-QAM with Gray encoded bits, constellation size  $M(\hat{\gamma})$ , and transmit power  $S(\hat{\gamma})$ , the instantaneous BER as a function of  $\gamma$  and  $\hat{\gamma}$  on an AWGN channel, is approximated by [6]

$$\begin{aligned} \text{BER}(\gamma, \hat{\gamma}) &\approx \frac{2}{\log_2 M(\hat{\gamma})} \left(1 - \frac{1}{\sqrt{M(\hat{\gamma})}}\right) \times \\ &\text{erfc}\left(\sqrt{1.5 \frac{\gamma S(\hat{\gamma})}{M(\hat{\gamma}) - 1}}\right) \end{aligned} \quad (17)$$

which is tight for high SNRs. In [1], it is shown that (17) can be further approximated as

$$\text{BER}(\gamma, \hat{\gamma}) \approx 0.2 \exp\left(\frac{-1.6\gamma}{M(\hat{\gamma}) - 1} \frac{S(\hat{\gamma})}{\bar{S}}\right) \quad (18)$$

which is tight within 1 dB for  $M(\hat{\gamma}) \geq 4$  and  $\text{BER} \leq 10^{-3}$ . By averaging (18) over the whole range of the instantaneous true SNR  $\gamma$ , the instantaneous BER as a function of the instantaneous predicted SNR is

obtained as

$$\begin{aligned} \text{BER}(\hat{\gamma}) &= \int_0^\infty \text{BER}(\gamma, \hat{\gamma}) f_\gamma(\gamma|\hat{\gamma}) d\gamma \\ &\approx \frac{a(\hat{\gamma})}{b(\hat{\gamma})} \exp\left(\frac{c(\hat{\gamma})^2}{4b(\hat{\gamma})}\right), \quad \hat{\gamma} \geq \bar{\gamma}\sigma_{\epsilon_c}^2 \end{aligned} \quad (19)$$

where

$$a(\hat{\gamma}) = \frac{0.2}{\bar{\gamma}\sigma_{\epsilon_c}^2} \exp\left(-\frac{\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2}{\bar{\gamma}\sigma_{\epsilon_c}^2}\right), \quad (20)$$

$$b(\hat{\gamma}) = \frac{1}{\bar{\gamma}\sigma_{\epsilon_c}^2} + \frac{1.6}{M(\hat{\gamma}) - 1} \frac{S(\hat{\gamma})}{\bar{S}}, \quad (21)$$

and

$$c(\hat{\gamma}) = \frac{2}{\bar{\gamma}\sigma_{\epsilon_c}^2} \sqrt{\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2}. \quad (22)$$

In Figure 2, the instantaneous BER is illustrated for the constant transmit power  $S(\hat{\gamma}) = \bar{S}$  and different constellation sizes where both (17) and (18) are used in (19). It is shown that the results based on the approximation given by (18) are tight enough.

## 5 Optimal Rate and Power Adaptation

The spectral efficiency of a modulation scheme is given by the average data rate per unit bandwidth ( $R/B$ ) where  $R$  is the data rate and  $B$  is the transmitted signal bandwidth. When a modulation with constellation size  $M(\hat{\gamma})$  is chosen, the instantaneous data rate is  $k(\hat{\gamma})/T_s$  (bps). Assuming the Nyquist data pulses ( $B = 1/T_s$ ), the spectral efficiency is given by

$$\eta_B = \frac{R}{B} = \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \quad \text{bps/Hz} \quad (23)$$

We would like to maximize the spectral efficiency subject to the average transmit power constraint as

$$\int_0^\infty S(\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \leq \bar{S} \quad (24)$$

and instantaneous BER constraint given by

$$\text{BER}(\hat{\gamma}) = P_b. \quad (25)$$

Under a BER constraint,  $S(\hat{\gamma})$  can be evaluated in terms of  $\hat{\gamma}$  and  $M(\hat{\gamma})$  as the following: Let us define

$$x = \frac{1}{\bar{\gamma}\sigma_{\epsilon_c}^2 b(\hat{\gamma})}, \quad 0 \leq x \leq 1 \quad (26)$$

and

$$y = \frac{\hat{\gamma}}{\bar{\gamma}\sigma_{\epsilon_c}^2}, \quad y \geq 1 \quad (27)$$

and rewrite (25) as

$$0.2xe^{(1-y)(1-x)} = P_b. \quad (28)$$

Taking the natural logarithm of (28) and then using the Taylor approximation  $\ln x \approx x - 1$  about  $x = 1$ , we obtain

$$x \approx 1 + \frac{1}{y} \ln\left(\frac{P_b}{0.2}\right) \quad (29)$$

and consequently

$$\begin{aligned} S(\hat{\gamma}) &\approx h(\hat{\gamma}, M(\hat{\gamma})) \\ &= \frac{-(M(\hat{\gamma}) - 1)}{1.6} \frac{\bar{S} \ln\left(\frac{P_b}{0.2}\right)}{\hat{\gamma} + \bar{\gamma}\sigma_{\epsilon_c}^2 \ln\left(\frac{P_b}{0.2}\right)} U(\hat{\gamma} - \bar{\gamma}\sigma_{\epsilon_c}^2). \end{aligned} \quad (30)$$

As explained in Section 2, a signal constellation and consequently a transmission rate is assigned to each predicted SNR region boundary. Therefore, for a given number of signal constellations, the optimum predicted SNR region boundary for each constellation corresponds to the optimum transmission rate, accordingly. Moreover, the transmit power can be obtained from the instantaneous predicted SNR based on (30). Therefore, the optimization problem can be simplified to finding only the optimal region boundaries. The Lagrangian function is given by

$$\begin{aligned} J(\hat{\gamma}_0, \hat{\gamma}_1, \dots, \hat{\gamma}_{N-1}) &= \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \\ &+ \lambda \left( \sum_{i=0}^{N-1} \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} h(\hat{\gamma}, M_i) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} - \bar{S} \right) \end{aligned} \quad (31)$$

where the optimal region boundaries are found by solving

$$\frac{\partial J}{\partial \hat{\gamma}_i} = 0, \quad 0 \leq i \leq N-1 \quad (32)$$

which results in

$$h(\hat{\gamma}_i, M_{i-1}) - h(\hat{\gamma}_i, M_i) = -\frac{k_{i-1} - k_i}{\lambda} \quad (33)$$

where  $k_{-1} = 0$  and  $M_{-1} = 1$  are assumed. Taking into account (30) and (33), the optimum region boundaries are obtained by

$$\hat{\gamma}_i = \ln\left(\frac{P_b}{0.2}\right) \left( \frac{\bar{S}}{1.6} \frac{M_{i-1} - M_i}{k_{i-1} - k_i} \lambda - \bar{\gamma}\sigma_{\epsilon_c}^2 \right) \quad (34)$$

where the Lagrangian multiplier  $\lambda \neq 0$  is numerically evaluated based on the power constraint given as

$$\begin{aligned} &\sum_{i=0}^{N-1} \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} h(\hat{\gamma}, M_i) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} = \\ &\rho \exp\left(\frac{\sigma_{\epsilon_c}^2}{r_g - \sigma_{\epsilon_c}^2} \left(1 + \ln\left(\frac{P_b}{0.2}\right)\right)\right) \times \\ &\sum_{i=0}^{N-2} (M_i - 1) (\text{Ei}(\rho\lambda P_i) - \text{Ei}(\rho\lambda P_{i+1})) + \\ &(M_{N-1} - 1) \text{Ei}(\rho\lambda P_{N-1}) \leq \bar{S} \end{aligned} \quad (35)$$

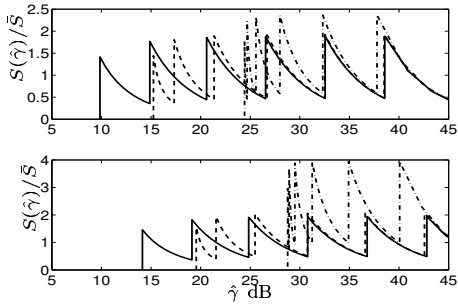


Figure 3: Optimum normalized transmit power versus average instantaneous predicted SNR of M-QAM schemes for  $\bar{\gamma} = 30$  dB. The upper and lower plots correspond to  $P_b = 10^{-3}$  and  $10^{-7}$ , and the solid, dashed and dashed-dotted lines correspond to  $\sigma_{\epsilon_p}^2 = 0.001$ ,  $0.01$ , and  $0.1$ , respectively.

where

$$\rho = \ln \left( \frac{P_b}{0.2} \right) \frac{\bar{S}}{1.6 \bar{\gamma} (r_g - \sigma_{\epsilon_c}^2)}, \quad P_i = \frac{M_{i-1} - M_i}{k_{i-1} - k_i} \quad (36)$$

and  $\text{Ei}(\cdot)$  is the Exponential Integral given by  $\int (e^x/x) dx = \text{Ei}(x)$ . When the optimal region boundaries are determined, the maximum spectral efficiency is evaluated through (16) and (23) as

$$\eta_B = \sum_{i=0}^{N-1} k_i \left[ \exp \left( - \frac{\max(\bar{\gamma} \sigma_{\epsilon_c}^2, \hat{\gamma}_i) - \bar{\gamma} \sigma_{\epsilon_c}^2}{\bar{\gamma} (r_g - \sigma_{\epsilon_c}^2)} \right) - \exp \left( - \frac{\max(\bar{\gamma} \sigma_{\epsilon_c}^2, \hat{\gamma}_{i+1}) - \bar{\gamma} \sigma_{\epsilon_c}^2}{\bar{\gamma} (r_g - \sigma_{\epsilon_c}^2)} \right) \right]. \quad (37)$$

## 6 Results

We assume six different M-QAM signal constellations corresponding to 4-QAM, 16-QAM, 64-QAM, 256-QAM, 1024-QAM and 4096-QAM, are available at the transmitter. Also, a flat Rayleigh fading channel with  $r_g = 1$  is presumed. The optimal region boundaries for the cases where the required instantaneous BERs are  $10^{-3}$  and  $10^{-7}$  and the average received SNR  $\bar{\gamma} = 30$  dB are evaluated and shown in Figure 3, for the prediction error variances  $\sigma_{\epsilon_p}^2 = 0.001$ ,  $0.01$  and  $0.1$ . It is observed that the optimal region boundaries are increased when the prediction error increases, as expected. The maximum spectral efficiency for  $P_b = 10^{-3}$  and  $10^{-7}$  and  $\sigma_{\epsilon_p}^2 = 0.001$ ,  $0.01$  and  $0.1$  are illustrated in Figure 4. The curves show that the gain in the spectral efficiency using good predictors is considerable as compared to the poor predictors. The reduction in spectral efficiency with an increasing prediction inaccuracy becomes larger when the required BER is increased. Compare  $P_b = 10^{-7}$  to  $P_b = 10^{-3}$  in Figure 4.

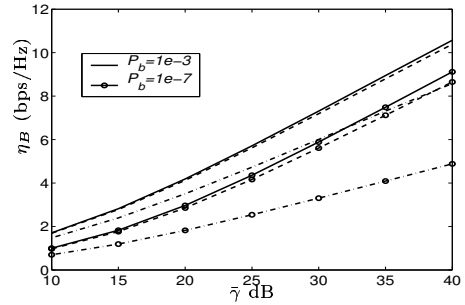


Figure 4: M-QAM Spectral efficiency versus average received SNR for  $P_b = 10^{-3}$  and  $10^{-7}$ . The solid, dashed and dashed-dotted lines correspond to  $\sigma_{\epsilon_p}^2 = 0.001$ ,  $0.01$  and  $0.1$ , respectively.

## 7 Conclusion

The optimum design of an adaptive modulation scheme based on M-QAM modulation is investigated. The transmitter adjusts the transmission rate and power based on the predicted SNR to maximize the spectral efficiency while satisfying the instantaneous BER and average transmit power constraints. The instantaneous BER formula as a function of predicted SNR, average SNR and predicted error variance is evaluated. Optimum solutions for the adaptive rate and transmit power are derived. The analytical results show that the predicted SNR boundaries for a given constellation size are increased when the prediction error increases. Moreover, the spectral efficiency decreases as the predictor error variance increases and (or) the required BER decreases, as expected. However the loss in spectrum efficiency can be significantly reduced by using a good predictor.

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