ON THE ROBUSTNESS AND PERFORMANCE TRADEOFFS FOR OFDM CHANNEL ESTIMATION

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ABSTRACT

This paper addresses the problem of designing a robust, VLSI oriented, pilot assisted channel estimation for OFDM receiver in the presence of frequency offset. In the absence of frequency offset, two channel estimation algorithms are studied: Lagrange and Wiener interpolators. The proposed schemes are used for designing DVB-T 2K mode channel estimation unit. For time interpolation, the experimental results show that the performance of the Wiener filter is far better than the performance of the Lagrange interpolator.

In order to derive a VLSI oriented channel estimation algorithm, the length of the filter must be optimized for a given performance (BER or MSE). In this paper, in the absence of frequency offset, performance tradeoff is described.

The frequency offset introduces ICI (Inter-Channel Interference) that further degrades the BER of the OFDM system. An accurate closed form expression for the ICI noise power is derived. Then, based on this formula, a robust Wiener is designed. The experimental results show that the performance of the robust Wiener filter is far better than that of the Wiener filter designed under perfect synchronization.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation very robust in multi-path fading environments[1]. Recently it has been proposed as a modulation scheme for broadband wireless and wired applications. These applications range from digital audio/video broadcasting known as DVB-T (Digital Video Broadcasting over Terrestrial channel) and DAB (Digital Audio Broadcasting) to high-data rate indoor wireless communication (WLAN) [2]. The basic idea of OFDM is to split the entire bandwidth into orthogonal sub-channels, and modulate each one separately. Thus one major issue in OFDM is to preserve orthogonality among adjacent sub-carriers. This is done by suitable selection of the spacing between adjacent sub-carriers. However, in reality even if the spacing has been optimally selected, the orthogonality conditions among adjacent sub-channels are not preserved due to frequency offsets and imperfect synchronization between the receiver and transmitter. In [4], it has been shown that the performance of the OFDM receiver is highly vulnerable to the synchronization error.

Over the last decade, synchronization of OFDM receivers has been the subject of extensive research, see *e.g.* [11, 10]. Most of the reported schemes for the case of coherent OFDM assume perfect estimates of the channel impulse response at the receiver.

In the last decade, channel estimation for coherent OFDM receiver has been studied extensively. In [5], 2x1D Wiener filter has been compared against 2D Wiener filter in terms of complexity versus performance. The author concluded that for a given complexity the 2x1D Wiener filter has similar performance to the 2D Wiener filter. In [13], the case of a mobile receiver has been considered, a channel estimation scheme based on 2x1D Wiener filter has been proposed. The Wiener filter for the frequency interpolation has been adapted based on the estimation of the delay spread of the channel.

Mismatch analysis between interpolation filter and the actual channel statistics has been analyzed in [7].

The aforementioned works have addressed the problem of designing the channel estimation units for OFDM mobile receivers under different assumptions. However, to the best of our knowledge there are still open questions that were not efficiently addressed by the aforesaid works. These questions are

1. What will be the effect of the frequency offsets on the performance of the channel estimators?
2. What is the robustness of the channels estimators

if the assumption regarding the statistics of the channel is different from the original channel?

3. Under these circumstances, what will be the way to derive a VLSI oriented channel estimation algorithm?

The rest of the paper is organized as follows. In Section 2 the problem is formulated and our work is put into perspective with the existing approaches. The channel estimation algorithms are reviewed in Section 3, a robust Wiener filter designed in the presence of frequency offset is described, and a closed-form expression for the ICI noise power is derived in Appendix-A. The experimental results are reported in Section 4. Finally the paper is concluded in Section 5.

2. CHANNEL ESTIMATION ALGORITHMS FOR OFDM RECEIVER

2.1. Problem formulation

Following the same notations used in our previous paper, the complex envelope of the OFDM signal is given by the following equation[6].

$$v(t) = \sum_{m=0}^{\infty} \sum_{k=K_{min}}^{K_{max}} c_{m,k} \exp\left(j2\pi \frac{k(t-mT_U)}{T_U}\right) h_a(t-mT_S), \quad (1)$$

where $\vec{c(m)} = [c_{m,K_{min}}, c_{m,1}, \ldots, c_{m,K_{max}}]^T$ is a column vector containing $K_{max} - K_{min} + 1$ complex source symbols at epoch m, $T_U = N \times T$ is the duration of the OFDM symbol, T is the duration of the complex symbol,
$c_{m,k}$,$N = 2^{\lceil \log_2(K_{max}-K_{min}+1) \rceil}$ is the size of the IFFT processor, $K = k - \frac{K_{max}}{2}$, K_{max} is the value of the carrier number K_{max},K_{min} is the value of the carrier number K_{min}, $h_a(t)$ is the amplitude pulse shaping signal of duration T_s, $T_S = T_U + \Delta$ is the OFDM symbol duration after cyclic prefix, Δ is the duration of the cyclic prefix. In the rest of the paper, K_{max} and K_{min} are fixed to $N-1$ and 0, respectively.
In the sequel, the expression of the received symbol, in the case of perfect and in the presence of frequency offset is reviewed.

2.1.1. *Perfect synchronization*

Under a perfect synchronization assumption, the received signal, $r(t)$, after sampling, is given by the following equation.

$$r_k(iT) = \sum_{j=0}^{N_L-1} h_k(jT) v_k((i-j)T) + \eta_k(iT), \quad (2)$$

where $h_k(jT)$ is the impulse response of the channel, $\eta_k(iT)$ is the Additive White Gaussian Noise (AWGN) sampled at instants iT, and N_L is the channel length.

After the FFT, the received complex symbols, $c^r_{k,n}$ is given by Eq.3.

$$c^r_{k,n} = c_{k,n} H_{k,n} + \zeta_{k,n}, \quad (3)$$

where $H_{k,n}$ is the frequency response of the channel given by $H_{k,n} = \sum_{i=0}^{N_L-1} h_k(i) \exp\left(-j2\pi \frac{in}{N}\right)$, and $\zeta_{k,n} = \sum_{i=0}^{N-1} \eta_k(i) \exp\left(-j2\pi \frac{in}{N}\right)$.
In order to recover the original complex symbols, $c_{k,n}$, $H_{k,n}$ must be estimated. The pilots are inserted, by the transmitter, in an ordered manner. The receiver uses these pilots to reliably estimate the channel using efficient interpolation schemes. The location and number of pilots depend on many factors including, the characteristics of the channel (Doppler and delay spread) and the characteristics of the OFDM (channel encoding, modulation and duration of the OFDM symbol)[5][13].

Generally speaking, the wireless medium (WM) is mathematically described by the following equation [9]

$$h(t,\tau) = \sum_i \gamma_i(t) c(\tau - \tau_i), \quad (4)$$

where $\gamma_i(t)$ and τ_i are the delay and complex amplitude of the ith path, respectively. The entity $c(\tau)$ is a pulse-shaping filter. The frequency response of the WM is given by Eq.5.

$$H(t,f) = \int_{-\infty}^{+\infty} h(t,\tau) e^{-j2\pi f\tau}. \quad (5)$$

The channel given by Eq.5 is a continuous WSS channel which is assumed to be band limited. The sampled channel is given by Eq.6

$$H[mT_s, k\Delta_f] = C(k\Delta_f) \sum_i \gamma_i(mT_s) e^{-j2\pi k\Delta_f \tau_i} \quad (6)$$

At this point it is necessary to mention that Eq.6 is the correct version of Eq.3 derived in [9].

In order to estimate the channel, the transmitter inserts pilots at known locations. The locations and the density of the pilots should allow for accurate estimation of the channel, as discussed earlier . The distance between the pilots must comply with the Nyquist Theorem. This issue is however beyond the scope of this work. The pilots are thus placed in a 2-D matrix. Fig.1 depicts an example of a received OFDM frames.

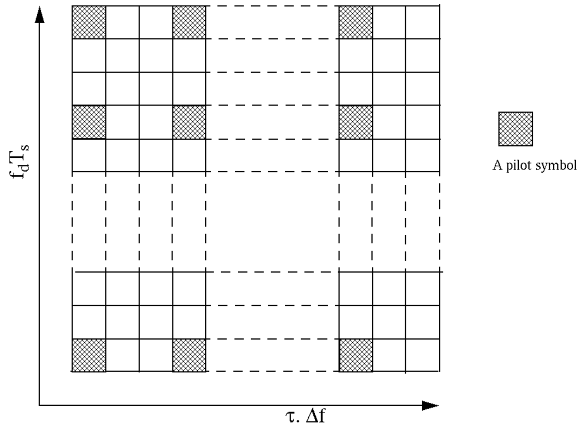

Figure 1: Locations of the pilots in OFDM frames.

2.1.2. In the presence of frequency offset

In the presence of frequency offsets the expression of the received symbol given by Eq.3 is not appropriate. Instead the appropriate expression for the received symbol is given by Eq.7

$$c^r_{k,n} = c_{k,n}H_{k,n}\left(\frac{\sin\pi\epsilon}{N\sin\pi\frac{\epsilon}{N}}\right)e^{j\pi\epsilon\frac{N-1}{N}} + I_{k,n} + \zeta_{k,n} \quad (7)$$

$$I_{k,n} = e^{j\pi\epsilon\frac{(N-1)}{N}}\sum_{m=0,m\neq n}^{N-1} c_{k,m}H_{k,m} \times \left(\left[\frac{sin(\pi\epsilon)}{Nsin(\frac{\pi(m-n+\epsilon)}{N})}\right] \times e^{-j\pi\frac{(m-n)}{N}}\right),$$

where ϵ is the relative frequency offset [10].

The fundamental difference between Eq.3 and Eq.7 is that the received complex symbol is further contaminated by a noise which is not only additive but also multiplicative. Additionally, its statistics depend on the frequency offsets. This noise deteriorates the performance of the channel estimation schemes.

2.2. Related work

The complexity constrained channel estimation in the absence of ICI noise is studied in [3]. The study of the robust channel estimation for OFDM systems is reported in [7] and [8]. In [7], *Li et al.* have showed that the performance of the MMSE estimator is not affected by the mismatch in time-correlation of the channel. However, it was found that the performance of the MMSE is affected by the mismatch in the frequency-correlation of the channel. A robust Wiener filter has been derived. This approach is more efficient than the adaptive filter approach proposed by *Sanzi et al* [13]. This is because the adaptive filter requires the adjustment of the filter coefficient based on the estimation of the delay spread of the channel. This re-adjustment of the filter coefficient requires hardware implementation of a unit that takes care of a complex matrix inversion, which surely comes at the expense of an increased receiver complexity and cost (power consumption and silicon area). In addition, the results presented in [7] show that the performance of the robust estimator is very close to the performance of the MMSE estimator designed to match the channel. The only penalty of this approach is that the filter length of the robust estimator is somewhat larger (50 taps) than that of the MMSE filter designed to match the channel(5 taps). In addition to this limitation, the model used by *Li at al.* assumes a perfect synchronization. This assumption is unrealistic in practical receiver. Moreover, the implementation complexity has not been addressed.

The performance of the channel estimation under the presence of frequency offset has been investigated by *Seung et al.* [8]. It was found that the performance of the channel estimator with frequency offset compensation is better than the case with no frequency offset compensation. This is because larger frequency offset may cause a great mismatch in the channel. The above paper is a good start to study the effect of the frequency offset on the performance of the channel estimation. However, still no closed form exists. In addition, no bounds for the tolerable frequency offset was investigated. This bound is very useful to the frequency offset compensation algorithm. Moreover, the complexity of the channel estimation has not been studied.

3. LINEAR INTERPOLATION

In Section 2, the problem of channel estimation is translated back into the problem of interpolation. The interpolation is a 2-D filtering process. For most of the wireless channel model the 2-D channel estimation can be efficiently done using 1-D channel estimation that is, firstly the time interpolation is executed (vertical direction in Fig.1), secondly, the interpolator is applied in the frequency direction (horizontal direction in Fig.1). In the following, we review two well known interpolators: Lagrange and Wiener. Let $f_n = \{f(k), k = 0, \ldots, N\}$ be a Stationary Discrete-time Stochastic Process (SDSP) obtained by sampling a continuous band-limited process $f(t)$ at time instant, $k \cdot T$, where $T = \pi/\sigma$, σ is such that $S(w) = 0$ if $|w| > \sigma$[15]. Let g_n be an SDSP obtained by decimating f_n by factor of M such that $g(k) = f(M \cdot k)$. The interpolation problem is the process of building up an estimator that accurately estimates f_n given g_n. If we denote by $\hat{f}_n$ the estimate of f_n given g_n, then the estimator should minimize, in a statistical sense, the error given by Eq.8

$$e_n = f_n - \hat{f}_n. \quad (8)$$

For a given k, such that k is not a multiple of M, polynomial interpolator uses N_u knots in the vicinity of k to estimate $f(k)$. Given the sequence $k = l \cdots M + q$, we define the N_u knots in the vicinity of k by the following sets $N_v = \{l - N_{left} - 1, l - N_{left}, \cdots, l\} \bigcup \{l + 1, l + 2, \cdots, l + N_{right}\}$. This technique can be seen as a moving window, where the window is centered at k and its elements are in N_v. A window of size 4, $N_{left} = 2$ and $N_{right} = 2$ is shown in Fig.2. The number of knots used to estimate f(k) is $N_u = N_{right} + N_{left}$.

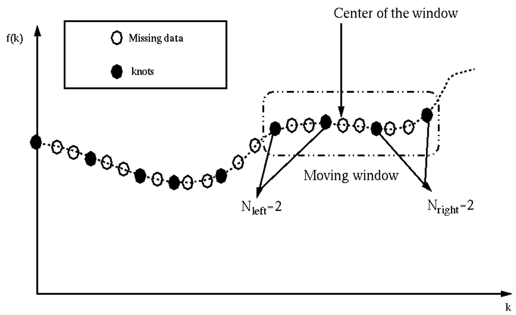

Figure 2: Polynomial interpolator using moving window.

The estimation of $f(k)$ is given by

$$f(\hat{k}) = \sum_{i=0}^{N_{left}-1} \beta_{l,i} M(l-i) + \sum_{i=1}^{N_{right}} \alpha_{l,i} M(l+i), \quad (9)$$

where $\beta_{l,i}$ and $\alpha_{l,i}$ are the polynomial coefficients. There are several algorithms that compute $\beta_{l,i}$ and $\alpha_{l,i}$ such as Lagrange, cubic-spline, Hermitian, and the least mean square interpolator[14].

In this paper, Lagrange and Wiener interpolators are considered. Lagrange and Wiener filters belong to the class of Linear filtering. The main difference between those two schemes is the way the coefficients of the interpolator are computed.

3.1. Lagrange interpolator

Let T_d be the sampling rate before interpolation and T_p be the sampling rate after interpolation. Let the window be centered at m. Lagrange interpolator used to estimate $f((m + \mu_k)T_d)$ is given by Eq.10, where $\mu_k = \frac{k}{L}$ for $k \in \{1, \ldots, L-1\}$ and $L = \frac{T_d}{T_p}$.

$$f((m + \mu_k)T_d) = \sum_{n=-N_{left}}^{N_{right}} f((m+n)T_d) \times \quad (10)$$

$$\Pi_{i=-N_{left}, i \neq n}^{N_{right}} \frac{\mu_k - i}{n - i}$$

The above equation can be seen as a filtering process using an FIR filter having the following taps,

$$h_\mu(n) = \Pi_{i=-N_{left}, i \neq n}^{N_{right}} \frac{\mu - i}{n - i}, n \in \{-N_{left}, \ldots, N_{right}\}. \quad (11)$$

3.2. Wiener interpolator

The Wiener filter theory takes advantage of the statistics of the sequences f_n in order to derive a filter that minimizes the mean square of the error given by Eq.8. Let the sequence $r(\mu T_d) \stackrel{\Delta}{=} E\left(f((n+\mu)T_d) f^\star(nT_d)\right)$ be the autocorrelation of $f(n)$ sampled at instants $(n + \mu)T_d$, where μ satisfies the following inequality $\frac{1}{L} \leq \mu \leq \frac{L-1}{L}$. In the rest of the derivation, the variable T_d is omitted from the equations.

In the noiseless case, the Wiener-Hopf equation suggests that, for a given μ_k, the optimum MSE interpolator of the sequence $\mathbf{f} = [f(n - N_{left} - 1), f(n - N_{left}), \cdots, f(n + N_{right})]^t$, is given by Eq.12

$$\mathbf{W}_\mu^k = R_{f,f}^{-1} \rho_\mu^k, \quad (12)$$

where $\rho_\mu^k = [r(-N_{left} - 1), \ldots, r(N_{right}]^t$, $R_{ff} \stackrel{\Delta}{=} E\left(\mathbf{f}\mathbf{f}^\mathbf{H}\right)$, and W_μ^k is the coefficient for the Wiener interpolator.

In the presence of noise, which is assumed to be additive, uncorrelated, and WSS, the received signal at the instant of time n, is given by Eq.13

$$f_{noisy}(n) = \Gamma f(n) + x(n) + y(n), \quad (13)$$

where $x(n)$ is an AWGN with zero mean and a variance σ_x^2. $y(n)$ is an uncorrelated WSS signal with zero mean and a variance σ_y^2. Γ is an attenuation factor which is independent of n. Let $\mathbf{f}_{noise}$ be the noisy vector of $\mathbf{f}$, then the Wiener filter acts as a smoother and interpolator. Based on our assumption regarding the statistics of the noise, we see that $\rho_{\mu,noise}^k = E\left(\mathbf{f_{noise}} \mathbf{f_{noise}^H}\right)$, satisfies the relation $\rho_{\mu,noise}^k = \Gamma \rho_\mu^k$. On the other hand the autocorrelation matrix $\mathbf{R_{f_{noisy}f_{noisy}}} = E\left(\mathbf{f}_{noisy} \mathbf{f_{noisy}^H}\right)$ satisfies the following relation

$$\mathbf{R_{f_{noisy}f_{noisy}}} = \Gamma \mathbf{R}_{ff} + \underbrace{(\sigma_x^2 + \sigma_y^2)}_{\sigma^2} \mathbf{I}, \quad (14)$$

where $\mathbf{I}$ is the identity matrix. Thus, in the presence of an additive noise, the optimum Wiener filter coefficients are given by Eq.15

$$\mathbf{W}_{\mu,noise}^k = \left(\mathbf{R}_{ff} + \sigma^2 \mathbf{I}\right)^{-1} \rho_\mu^k \quad (15)$$

The noise power for the ICI noise is derived in Appendix-A.

4. EXPERIMENTAL RESULTS

In order to validate our approach to answer the questions described in the introduction (Cf. Section 1), the digital video broadcasting (DVB-T) has been used as a case study[17]. This study is a step toward completing hardware implementation of the DVB-T receiver[16]. In the DVB-T receiver, the parameters such as location of the pilots, number of carriers, OFDM symbol duration, length of the cyclic prefix (Δ), and the signal space diagram are fixed. DVB-T supports the following modulation: QPSK, 16-QAM and 64-QAM. Each signal space diagram has two possible configurations: Uniform if $\alpha = 1$, non-uniform if $\alpha = 2$ or $\alpha = 4$. At the receiver side, the received complex symbol (after channel correction) is efficiently demapped using the *MUSCOD* algorithm described in [16].

In our experiments, the WSSUS channel has been used to model the WM [12]. In addition, 2K mode DVB-T with the cyclic prefix of length (Δ) equals 1/32 and a uniform modulation ($\alpha = 1$) have been used. It is also assumed that the maximum delay spread of the channel (D_{max} in WSSUS channel model) equals the cyclic prefix length. The BER and the SER were obtained using the Monte-Carlo simulation techniques as described in [18].
The simulated BER in presence of an AWGN channel is depicted in Fig.3.

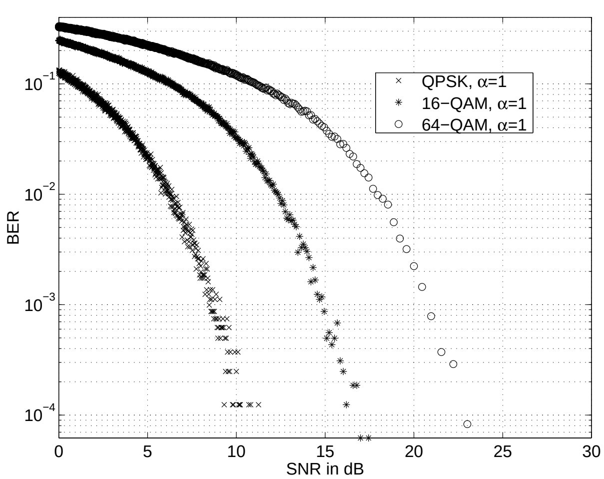

Figure 3: Simulated BER for DVB-T, 2 K mode, uniform modulation, in the presence of AWGN.

4.1. Perfect synchronization

In order to estimate the implementation complexity, the number of multiplications and memory consumptions are assumed to be proportional to the filter length. Although, in [19], it was shown that the computational complexity is inadequate metric to compare the VLSI implementation of different channel estimation algorithms (different filter length). This step is however necessary towards derivation of a VLSI oriented algorithm. The memory consumption tradeoff is related to the size of the time interpolator. Because the receiver must keep the previously received OFDM frames in a memory. These frames are used to estimate the pilots in the vertical direction shown in Fig.1.
For real time application such as the DVB-T the time interpolator must be causal. However, for frequency interpolation, the causality of the filter is irrelevant. This is because after the FFT processor, the complex symbols of the same OFDM frame, are available simultaneously. In this section, the filter described by $L \times M$ means that $N_{left} = L$ and $N_{right} = M$ and the filter length is $N_{left} + N_{right}$. Thus the filter is causal iff $M = 0$.

Fig.4 shows a comparison between Wiener and Lagrange interpolators for the same filter length. From this figure it is clear that 3x0 Wiener filter (causal) has almost similar performance to the 3x3 Wiener filter. In addition, the figure shows that Lagrange interpolator has a very high MSE compared to the Wiener filter. This is because the filter coefficients for Lagrange interpolator are independent of the statistics of the channel.

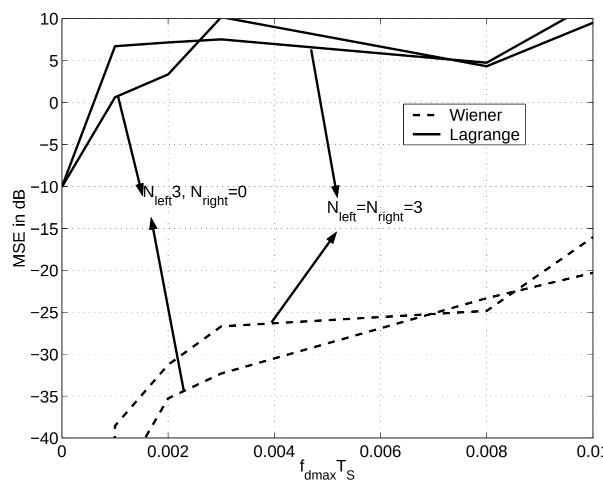

Figure 4: Comparison between Lagrange and Wiener for time interpolation.

Fig.5 shows the performance of the Wiener filter as a function of the filter length and the SNR. The result show that in this particular WSSUS configuration, 3x3 Wiener filter has a very competitive performance, with lower computational complexity, compared to Wiener filter with higher length.

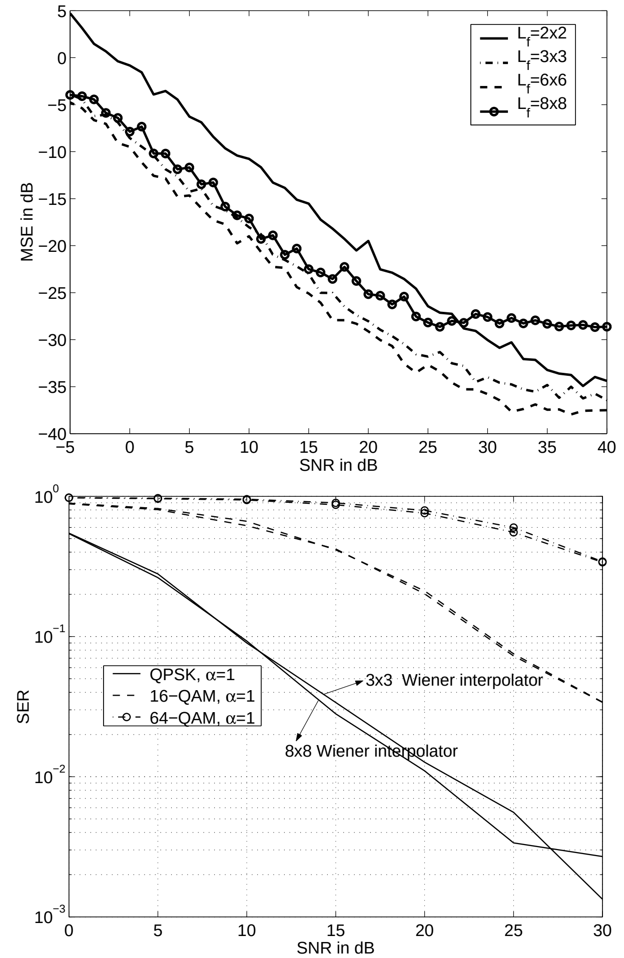

Figure 5: Complexity tradeoff for frequency interpolation.

4.2. In the presence of frequency offset

In order to assess the robustness of the Wiener filter, our first task is to evaluate the accuracy of Eq.24. In order to achieve this goal, the exact power of the ICI noise has been obtained using statistical simulation. The obtained value has been compared against Eq.24 and Eq.20. The results are summarized in Fig.6. The curves clearly show that Eq.24 is very close to the exact power compared to Moose's approximation given by Eq.20. The figure shows that Moose equation is indeed a tight upper bound of the ICI noise power, $\forall|\epsilon| < 0.5$. In order to quantify the impact of the frequency offset on the robustness of our Wiener filter, both Wiener filter and robust Wiener filter designed for an SNR equals 20dB have been used for frequency interpolation. Fig.7 depicts the curve of SER as a function of SNR in the presence of frequency offset, $\epsilon = 0.02$. The dashed curves are the SER for the robust Wiener filter. For

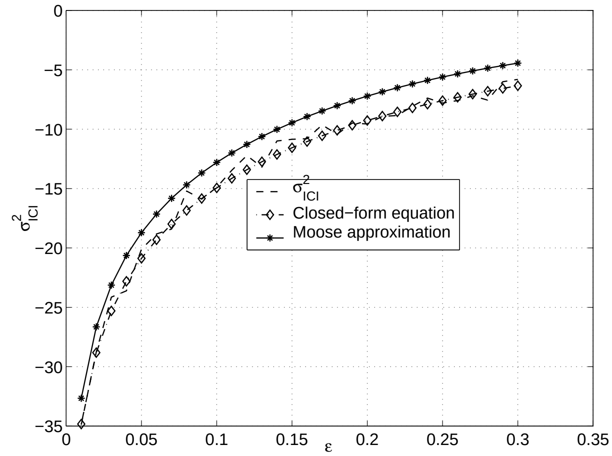

Figure 6: Simulated versus estimated σ^2_{ICI} using Eq.20 and Eq.24.

the sake convenience, the SER in the absence of frequency offset and for 16-QAM signal constellation is plotted in the same figure.

Fig.8 shows the SER for both robust an unoptimized Wiener filter designed for an SNR equals 20dB and in the presence of a frequency offset equals 0.1. Thus, from this results, it is clear that the Wiener filter designed under the presence of frequency offset has always lower SER than the Wiener filter designed under perfect synchronization.

5. CONCLUSIONS

In this paper Lagrange and Wiener filter for OFDM channel estimation have been compared. The comparison is done based on the achievable performance for a given SNR. For the same filter length, it was found that the performance of the Wiener filter is far better than that of the Lagrange interpolator. In addition, a performance tradeoff to select the Wiener filter length has been reported. For the case of DVB-T, 2K mode it was found that 3x3 Wiener for frequency interpolator has a competitive performance for a given complexity compared to the Wiener with higher filter length.

The above channel estimators have been designed under a perfect synchronization assumption. However, it was found that the frequency offset introduces ICI noise that degrades the performance of the interpolators. In order to tackle this problem, a closed form expression for the power of the ICI noise has been derived. It was found that this closed form is very accurate compared to the closed form expression derived by Moose [10]. This closed form has been used to design a robust Wiener filter.

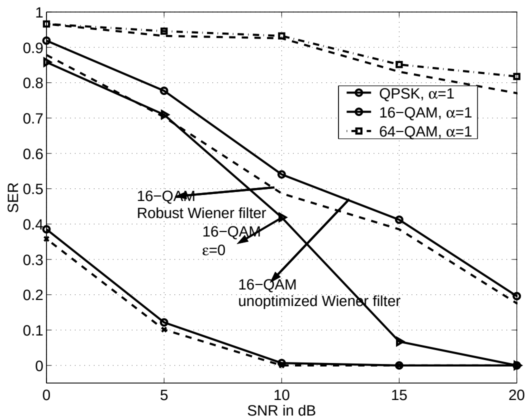

Figure 7: Comparison between Robust Wiener and Wiener filter designed for $SNR = 20dB$ and $\epsilon = 0.02$.

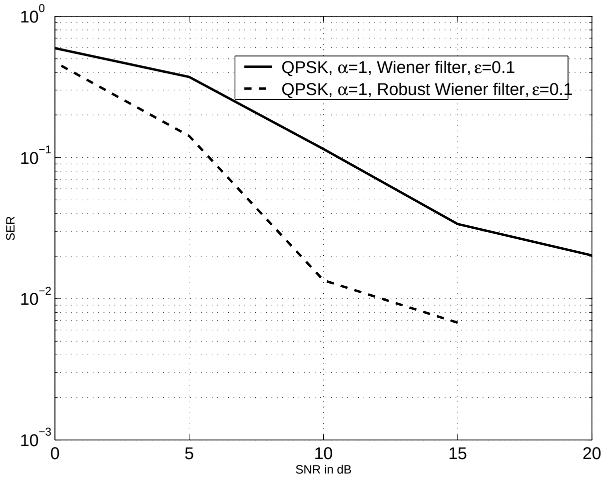

Figure 8: Comparison between Robust Wiener and Wiener filter designed for $SNR = 20dB$ and $\epsilon = 0.1$.

The outcome of the experimental results showed that the Wiener designed in the presence of frequency offset has lower BER than the Wiener filter designed under zero frequency offset assumption.

The course of this work is to design a VLSI oriented channel estimation algorithm. Toward achieving this goal, the effect of the quantization error as well as the implementation issues of the Wiener interpolator (such as algorithm strength reduction and low-power architecture synthesis) will be addressed in the future work.

Appendix-A

The aim of this appendix is to derive a closed-form expression for variance of $I_{k,n}$ given by Eq.16.

$$\sigma_{ICI}^2 = E\left(I_{k,n} I_{k,n}^{\star}\right). \qquad (16)$$

Now, assuming that the complex symbols, $c_{k,n}$ are uncorrelated with zero mean and variance equals one. In other words $E(c_{k,n}c_{l,m}^{\star}) = \delta(k-l)\delta(m-n)$. Following this assumption, it is easy to see that $E(I_{k,n}) = 0$. If $I_{k,n}$ is substituted for by its expression given by Eq.8, we obtain Eq.17.

$$\sigma_{ICI}^2 = E\left(\sum_{l=0,l\neq n}^{N-1}\left(\sum_{m=0,m\neq n}^{N-1} c_{k,m}c_{k,l}^{\star}\beta_m^n\beta_l^{n^\star}H_{k,m}H_{k,l}^{\star}\right)\right), \qquad (17)$$

where $\beta_j^n = \left[\frac{sin(\pi\epsilon)}{Nsin(\frac{\pi(j-n+\epsilon)}{N})}\right] \times e^{-j\pi\frac{(j-n)}{N}}$. Using the linearity properties of the expectation and the assumption regarding the statistics of the symbols $c_{k,n}$, Eq.17 is reduced to Eq.18.

$$\sigma_{ICI}^2 = \left[\frac{sin(\pi\epsilon)}{N}\right]^2 \sum_{m=0,m\neq n}^{N-1}\left(\frac{E(|H_{k,m}|^2)}{sin\left(\frac{\pi(m-n+\epsilon)}{N}\right)^2}\right). \qquad (18)$$

The inspection of Eq.18 reveals that if the channel has a constant energy, then the power of the ICI noise, σ_{ICI}^2, is independent of the index k, however, this noise power is still dependent on the carrier frequency n. Assuming that the channel has a constant energy, which is normalized to one, the entity σ_{ICI}^2 can be accurately approximated by the following equation

$$\sigma_{ICI}^2 = \left[\frac{sin(\pi\epsilon)}{N}\right]^2 \sum_{m=0,m\neq n}^{N-1}\left(\frac{1}{sin\left(\frac{\pi(m-n+\epsilon)}{N}\right)^2}\right). \qquad (19)$$

The formula for the noise power given by Eq.19 depends on the carrier index n. This dependency makes the derivation of the Wiener filter dependent on the carrier index, which is far from being an acceptable if the algorithm is to be VLSI implemented. In [10] an upper bound for σ_{ICI}^2, which is independent of n, was found to be

$$\sigma_{ICI}^2 \leq 0.5497 sin\left(\pi\epsilon\right)^2, \forall|\epsilon| \leq 0.5. \qquad (20)$$

In order to derive an equation for the noise power which is independent of the carrier index, we assume that the frequency error, ϵ, after frequency offset estimation (compensation) is very small. Under this condition, the function $\sin(x)$ can be accurately approximated by its first order Taylor series, that is, $\sin(x) \approx x$. By doing so, Eq.18 becomes

$$\sigma_{ICI}^2 \approx \left(sin(\pi\epsilon)\right)^2 \underbrace{\sum_{m=0,m\neq n}^{N-1}\frac{1}{\left(\pi(m-n+\epsilon)\right)^2}}_{\Sigma_n}. \qquad (21)$$

In the sequel, we focus on finding a closed-form expression for Σ_n. If we change the variable from m to k such that $k = m - n$, the expression for Σ_n becomes

$$\Sigma_n = \pi^{-2} \left(\sum_{k=-n}^{N-1-n} (k+\epsilon)^{-2} \right). \qquad (22)$$

For relatively small ϵ, Σ_n can be approximated by the following equation

$$\Sigma_n \approx \frac{1}{3} + \frac{2}{90} (\epsilon\pi)^2 + \frac{2}{945} (\epsilon\pi)^4 . \qquad (23)$$

Thus, for small values of ϵ, the closed-form expression of the power noise is given by Eq.24

$$\sigma^2_{ICI} \approx \left(\frac{1}{3} + \frac{2}{90} (\epsilon\pi)^2 + \frac{2}{945} (\epsilon\pi)^4 \right) (sin(\pi\epsilon))^2 . \quad (24)$$

This expression is less than the upper-bound given by Eq.20.

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